Volume XXV. *September, 1907.* *Number 3.*

THE

PHYSICAL REVIEW.

HEAT OF EVAPORATION OF WATER.

By Arthur Whitmore Smith.

Introduction.

THE amount of heat required to evaporate a gram of water at ordinary room temperatures is only approximately known, and this knowledge comes largely from determinations made at other temperatures. The familiar formula of Regnault certainly gives too large values as is shown by the more recent work of Griffiths and of Henning for the range 30°–100° C., and of Dieterici at 0° C. The present investigation fills in the range 0°–40° C. And the need of this determination is evident when one considers that during the past ten years there have been in progress in this country several elaborate investigations, costing tens of thousands of dollars, and in each of which the heat of evaporation of water at room temperatures is an essential factor. For want of better values those of Regnault are still used.

The evaporation of water at low temperatures can be accelerated in two ways. The method usually employed is to reduce the pressure until the water boils freely at the desired temperature. The other method, which has been tried by several investigators and discarded as unsatisfactory, is to pass a stream of dry air through the water. This air becomes saturated with water vapor and the amount of evaporation can be easily controlled by regulating the air current. While there are precautions which must be observed, there are also advantages in the latter method, for not only is the amount of evaporation always under control, but, what is more im-

portant, the temperature of the water can be held constant at the desired point for any length of time. Moreover the stirring is complete and thorough without introducing any of the uncertainties or corrections involved with a mechanical stirrer. The evaporation of water in nature is slow and quiet, and takes place into air at atmospheric pressure. Natural conditions are therefore more nearly approached by this method than by violent ebullition under reduced pressure, and with the arrangement described below it has proved entirely satisfactory.

PLAN OF THE INVESTIGATION.

The plan of this investigation is to draw dry air through the water within a suitable calorimeter, thereby causing it to evaporate, and by means of an electric current to supply the heat required for this evaporation. As a matter of fact the heating current was always maintained constant, while the current of dry air was continually adjusted to the value required to maintain the reading of the thermometer at the chosen point. In this way all of the heat supplied by the current was used in the evaporation of water, and none of it went to changing the temperature of the apparatus. Moreover, as the calorimeter was always kept at the temperature which it assumed of itself after standing two or three days surrounded by a constant temperature bath, there is also no radiation correction. All that it is necessary to know is the amount of heat supplied by the electric current, and the amount of water evaporated.

The former quantity is readily computed from the formula EIT. For determining the latter, the various forms of calorimeters were all designed with the intention of directly weighing them on a delicate balance, just before and immediately after each experiment. The loss in weight would give at once the amount of water evaporated. However desirable such an arrangement might be it was found that the thermal uncertainties introduced by removing the calorimeter from its constant temperature chamber were greater than any possible gain from the direct weighing. It therefore became necessary to determine the amount of water evaporated by collecting it from the air current after it had left the calorimeter. The air was taken from the room and passed through two H_2SO_4

tubes, a tube filled with absorbent cotton to catch any possible acid that might by any chance come over with the air, then through a meter of block tin pipe in the constant temperature bath, to the calorimeter. Leaving the calorimeter, the air current, now laden with its load of moisture, passed as directly as possible to two H_2SO_4 tubes and thence to the aspirator pump. All the water brought from the calorimeter is left in these last two tubes, and their gain in weight gives the amount of water evaporated.

Advantages of This Method.

This method possesses several advantages over others that might have been used. The water is slowly evaporated into air at nearly atmospheric pressure and therefore corresponds more nearly to natural evaporation than when the water boils under reduced pressure. Besides water does not boil easily or steadily at these low temperatures. But the principal advantage is that an experiment can be commenced or ended at any time without disturbing the setup in the least, and one experiment can follow another with no interval between. When everything is running smoothly and well, a recently weighed pair of H_2SO_4 tubes can be inserted in the air circuit to collect the water vapor coming from the calorimeter, and the old tubes removed and weighed. Two hours later the tubes are changed again, and the collected water is that which has been evaporated by the heat supplied during the same period. Thus the experiment has neither starting nor stopping, with all the attending uncertainties and corrections.

The Calorimeter.

The final form of the calorimeter consisted of a large test tube, *E*, Fig. 1, closed at the top by a cork which was filled and covered with pure paraffin. Two glass tubes for the air current passed through this cork, one extending to the bottom of the test-tube, the other passing through only two centimeters. A third tube, closed at the lower end, and not shown in the figure contained the heating coil, a single heater from a Nernst lamp provided with current and potential terminals. No heat could escape up this tube as it was filled with beeswax the greater part of its length, which also served to support and insulate the four leading-in wires. The coil itself was

surrounded with kerosene, the better to conduct the heat to the surrounding water. A fourth tube of thin test-tube glass, also closed at the lower end and extending to the bottom of the calorimeter, contained the thermometer, *T*. Good thermal contact between the walls of the tube and the bulb of the thermometer was secured by means of a little mercury, sufficient to cover the latter. Thus the thermometer could be easily removed for comparisons and there was less chance of breakage, either of the thermometer or the calorimeter. These tubes were all sealed in at the top of the calorimeter and outside the paraffined cork by means of "universal wax," which while affording little support does ensure an air-tigh tclosure.

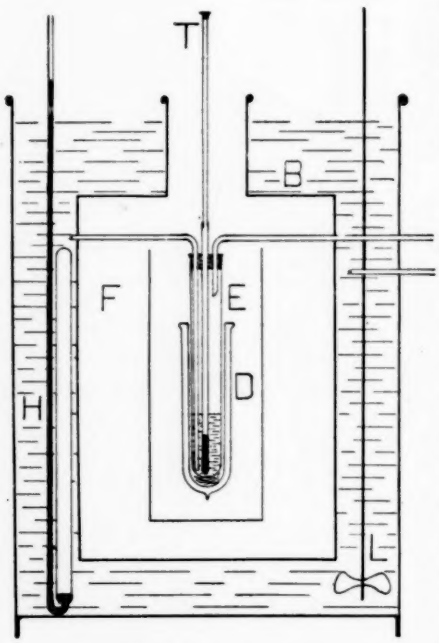

Fig. 1. The Calorimeter as arranged within the constant temperature water-bath.

This test-tube calorimeter stood within a double walled Dewar vacuum tube, *D*, the whole being well surrounded with light cotton wool and held in a pasteboard box, which in turn was supported in the middle of the constant temperature chamber, *F*. The remaining space around the sides, the top and the bottom of this chamber was lightly filled with cotton wool to prevent convection air currents.

The calorimeter was always packed thus thoroughly at least as long as the day before an experiment was to be performed in order that the entire contents of the chamber might assume the temperature of the surrounding bath. Heat moves but slowly through this thorough packing, but complete thermal equilibrium is essential if the calorimeter is to lose or gain no heat by radiation, convection or conduction during an experiment.

The Water-bath and Thermostat.

The constant temperature chamber was surrounded on all sides and nearly to the top of the neck by the constant temperature water bath, *B*. The temperature of this bath was maintained constant by an automatic gas thermostat, *H*, and the water was kept well stirred by a rotary stirrer, *L*, driven by a small electric motor. The final form of thermostat, and one which proved entirely satisfactory, consisted of a long glass bulb holding about one third of a liter, from the bottom of which the stem extended to above the surface of the water. Near the top of the stem it was drawn down to a small cross-section through which the mercury would rise or fall more rapidly than at other points where the tube was full size. The bulb was filled with alcohol for trial and as this proved satisfactory it was not exchanged for toluol. The stem, of course, was filled with mercury. Within the upper portion of the stem and extending down to the top of the mercury column was a smaller tube, ending in a fine point. Down this fine tube and up around it flows the gas for the burners under the bath. When the mercury rises it closes the end of this tube and thus shuts off the supply of gas. Much to my surprise this thermostat proved to be fully as sensitive at points where the stem was full size as at the more constricted portion. At the latter point the mercury rose rapidly, completely shutting off the supply of gas, save for the pilot light, and allowing the bath to cool — often far below the standard temperature. Where the tube was broader the mercury rose more slowly, shutting off the gas only partially and keeping a moderate, steady flame under the bath, just sufficient to maintain the constant temperature. In every way this thermostat did its work well, and it was not unusual for it to hold the bath constant to within one hundredth of a degree for a period of several consecutive days.

The H_2SO_4 Tubes.

The tubes containing the acid were obtained from the chemical laboratory, and are known as gas wash bottles. In each were placed 50 c.c. of concentrated, chemically pure, sulfuric acid, density 1.84, filling it to a depth of about six centimeters. The air bubbled through this in four small streams, and only after several grams of

water had been caught did any appreciable amount pass through to the second tube. When this stage was reached the tube was refilled with fresh acid.

Although this furnished a convenient method for collecting the water from the air current, considerable difficulty was experienced in weighing the tubes. That is, the same tube standing idle would not give a constant weight when weighed several times during the day. It is not necessary to recount here the weeks spent in trying to eliminate this variation. The trouble seemed to be due to the condensed layer of moisture on the outside of the glass, and various schemes to reduce this were tried, such as using an atmosphere dried with sulfuric acid; wiping the tubes with a soft cloth and allowing them to stand a definite period before weighing, etc. Finally, by placing one H_2SO_4 tube on one balance pan and a similar tube also filled with acid on the other pan, and treating them just alike between weighings, their difference was found to remain constant. No other counterpoise of brass or of glass would answer as well, and therefore with each pair of tubes was a third one through which no air ever passed, but which was otherwise treated precisely like the others, the three being kept together all the time whether in use or not. Of course they were all kept tightly closed from the air except the two when the air current was passing through them. The closure was effected by using short lengths of soft rubber tubing, the other end being plugged with a short piece of solid glass rod.

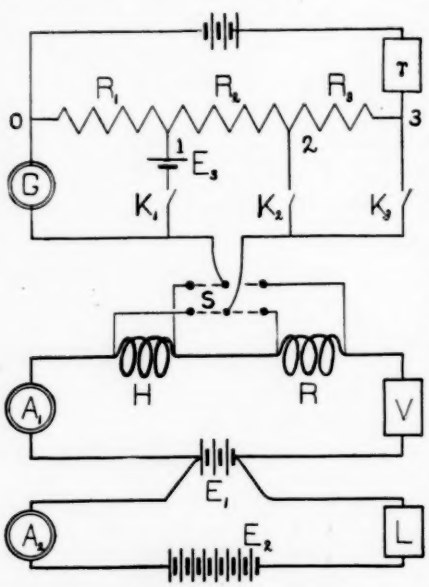

Fig. 2. Showing the electrical setup for measuring the energy supplied to the calorimeter.

Electric Current for Heating.

Arrangement and Measurement. — Fig. 2 shows the arrangement for measuring the energy supplied to the calorimeter by the electric current. The three storage cells, E_1, furnished the heating current, which was approximately measured by the ammeter, A_1. From A_1 the current passed through the heating coil, a standard manganin resistance, and an adjustable resistance, all in series. In order to keep the E.M.F. of E_1 as nearly constant as possible it was joined to eight cells, E_2, of the laboratory storage battery and sufficient resistance introduced at L to make the current at A_2 very nearly equal to that at A_1. Thus the battery, E_1, is continually charged with practically the same current it is supplying, and consequently its E.M.F. is extremely constant. This arrangement might be looked upon as having the main battery at E_2, while E_1 is an idle battery, "floating across the line" and serving to equalize all the irregularities of the current. Whichever view is taken, the resultant current is much more constant than could be obtained from the use of either battery alone.

The value of this current is determined by measuring the fall of potential over the standard resistance, R. The upper portion of the diagram represents a potentiometer arrangement. R_1, R_2, R_3 are certain resistances in a high grade decade Post Office box made by Nalder Bros. R_2 includes all the resistance from 0 to 2, and R_3 is all from 0 to 3. r is a decade 10,000-ohm box which can be varied as required to keep the potentiometer current, i, constant and at such a value that the fall of potential, $R_1 i$, over R_1 equals the E.M.F. of the standard cell, E_s. That is,

$$E_s = R_1 i. \qquad (1)$$

This equality is assured when the closing of K_1 produces no deflection of the galvanometer, G. When the double throw switch, S, is thrown to the right, the fall of potential, RI, over the standard resistance, R, is introduced into the galvanometer circuit. If upon closing the key, K_2, there is no deflection of the galvanometer it means that RI is just balanced by the fall of potential $R_2 i$ of the potentiometer circuit. Or

$$RI = R_2 i. \qquad (2)$$

Substituting the value of i from (1) above,

$$I = \frac{E_s R_2}{R R_1}. \tag{3}$$

The observer having charge of these measurements keeps close watch on this deflection by closing K_2 every few moments and the heating current, I, is maintained at just the value which gives this balance. This adjustment is readily made by means of a copper sulfate liquid resistance in parallel with a portion of the variable resistance. The minimum deflection read corresponded to a change of 1 part in 15,000 in the current. Greater changes than this were noted and the current adjusted to the proper value. When S is thrown to the left the fall of potential, E, over the heating coil is introduced into the galvanometer circuit. R_3 is then adjusted to the value which will give no deflection when K_3 is closed. That is,

$$E = R_3 i = E_s \frac{R_3}{R_1} \tag{4}$$

For every change made in R_3 there is, of course, a corresponding change made in r in order to keep the potentiometer current, i, constant. When once R_1, R_2 and R_3 have been adjusted to the proper values no further change is necessary as long as the resistance of the heating coil does not change. The entire work in this part of the experiment is to keep the heating current constant. Once in five minutes the keys K_1, K_2, K_3, are each closed and the corresponding galvanometer readings recorded. If these are not always strictly zero they will be sometimes positive and sometimes negative giving an average of nearly zero for each interval of an hour or two.

The Standard Resistance. — This was a coil of heavy manganin wire wound on a large brass tube after the fashion of the Reichsanstalt standards, the whole being suspended in a permanent bath of paraffin oil. Its resistance was determined by the Carey Foster bridge, and is expressed in terms of the mean of two standard 10-ohm coils made by Otto Wolff and bearing the seal of the Reichsanstalt. As the coil was about 4.63 ohms this comparison could not be made directly, but was obtained by placing the two standard

coils in parallel on the bridge, thus forming a resistance of five ohms with which the manganin coil could be compared directly. The mean of several determinations gave 4.6315 ohms for this resistance at 15° C. and it remained the same as the temperature was raised to 25° C.

The Standard Cell. — The standard cell used in these determinations was a cadmium cell of the *H* pattern and half filled with crystals of cadmium sulfate. Its E.M.F. was 1.01888 international volts at 20° C., that is when expressed in terms of 1.43400 volts for the Clark cell at 15° C. The cell was kept in a bath of kerosene oil and at about 20° C. For slightly different temperatures the value used for its E.M.F. is

$$E_t = 1.01888 - 0.00004\,(t - 20).$$

For this comparison I am indebted to Dr. W. D. Henderson who was working with the potentiometer at the time and using Professor Carhart's cell No. B_7. This cell has maintained a constant E.M.F. for several years, during which time it has been frequently compared with Clark cells which were carefully set up according to the regular specifications. Its E.M.F. in terms of these Clark cells is therefore very well known, and all comparisons made with it are in reality referred to the Clark cells as the ultimate standard.

Measurement of Time. — The beginning and close of each experiment was on the even minute as read from the second hand of an Elgin watch. No attempt was made to regulate this watch to keep exact time, but shortly after each observation it was compared with the standard clock which was gaining less than a second a day. Thus all of the recorded intervals of time are referred to this standard clock. In a former paper[1] it is shown that the uncertainty in time thus measured is about a tenth of a second; but if it were larger the error thus introduced would be insignificant as one experiment begins when the preceding one ends, and if the recorded duration of one is too long that for the following one will be too short by the same amount. Thus when the average is taken such errors disappear. At the beginning and ending of the day's work there is no such check, but an error of one whole second in eight hours would be insignificant.

[1] PHYS. REV., Vol. XVII., p. 214, Oct., 1903.

CORRECTIONS.

Heat Capacity. — During each experiment the temperature was maintained as nearly constant as possible. There were, however, occasional variations sometimes as great as two or even three hundredths of a degree, but no experiment was brought to a close if the temperature differed from that at the start by as much as one hundredth of a degree. Inasmuch as these slight variations were as often one way as the other, and as no heat was permanently stored in the apparatus through raising its temperature, there is no correction under this head.

Radiation. — As explained above, the outside bath was automatically maintained at a constant temperature, variations of one hundredth of a degree being rare. The calorimeter within the inner chamber will come to a constant temperature, and after standing for one or two days it is safe to assume that it will remain in thermal equilibrium with its surroundings during the short period of an experiment. This being the case, there is no gain or loss of heat by radiation and no correction under this head.

Conduction, Convection, Etc. — All the space about the calorimeter is filled with cotton wool which reduces any circulation of air to a minimum. The only metallic connections with the outside are the four small leading-in wires and the heat conducted in by them is inappreciable.

Starting and Stopping. — In all of the experiments here reported there is practically no starting and no stopping. When the calorimeter is holding a constant temperature with the air current bringing away its steady stream of water vapor and the electric current supplying the equivalent amount of heat, an experiment, so-called, can be made at any time that is convenient. An experiment is really only a single determination of the *time rate* of this stream of vapor, and several such determinations can be made in one day. For this purpose the two H_2SO_4 tubes are inserted in the outcoming air current for a measured interval of time. When these tubes are removed fresh ones are put in their place, and no vapor escapes unmeasured. Such a series of successive determinations are more valuable than the same number of experiments made at different times, because whatever thermal uncertainties may be left at the end

of one run are carried forward to the next. For example, if some part of the calorimeter should be warmer at the close of an experiment than it was at the beginning thereby holding heat which should have been used for the evaporation of water, and during the next run when equilibrium is attained the extra evaporation makes the collected water too large, then the average of these two results will not only possess the usual weight of a mean but it will be an absolute correction of this particular kind of uncertainty. It is for this reason that the experiments are made consecutive, one beginning where the other left off, and a set of four separate determinations made each time.

Stirring. — The water within the calorimeter was vigorously stirred by the passage of the air through it. Ordinarily this stirring would warm the water and necessitate a correction. In this case, however, the bubbles of air are cooled by expansion as they rise through the water. This warming of the water and cooling of the air balance each other and produces no resultant change in temperature.

That this balance is strictly exact in this calorimeter is shown by the following test. The calorimeter was filled to the usual depth with concentrated sulfuric acid, and allowed to assume thermal equilibrium with its surroundings and the water bath by standing for twenty-four hours. The usual air current was then drawn through the calorimeter for four hours. Temperature readings taken every ten minutes varied by a few thousandths of a degree, the maximum range being less than two hundredths. As closely as could be read, the final temperature was the same as the initial reading. It was interesting to watch the conflict between the cooling and warming, for while the thermometer always read about the same the mercury never was still, but was rapidly rising and falling through a range of three or four thousandths, apparently as the bubbles of expanding air played around the bulb of the thermometer or allowed the water to come in contact with it.

The Air Current. — If the air entering the calorimeter does not have the same temperature as the water within, the thermal balance will be destroyed as soon as the air current is started. Before entering the calorimeter the air passes through a long block tin

tube in the bath which warms it to the proper temperature. That no heat or cold is thus carried into the calorimeter by the air current is abundantly shown by the test experiment described above and in which no change in temperature could be detected during the four hours that the air was passing through the calorimeter.

Use of the Thermometer. — During an experiment all the temperature readings were taken on a Beckman thermometer, set so that the mercury stood near the top of the scale where it could be seen and read as only about one degree of the scale projected above the water bath. This thermometer served as an indicator only, and by its aid the temperature of the water in the calorimeter was maintained constant. The actual value of this temperature was determined by a direct comparison of this thermometer at the point where it was used with a standard thermometer, B.S. No. 2,107, which had been recently calibrated by the Bureau of Standards. The corrections then found are used in reducing its readings to the actual temperature.

THE EXPERIMENTS.

Preparation. — As already noted, four consecutive experiments were made at one time. In preparation for each set the calorimeter was filled with fresh distilled water and placed in readiness within the constant temperature chamber. If its temperature differed more than a few tenths of a degree from that of the chamber it was brought nearly up by the electric current. In this condition it was left until the next morning, by which time it reached a stable equilibrium with its surroundings. The proof of this equilibrium is not that the thermometer in the calorimeter reads the same as that in the bath, for this might be so and the calorimeter continue to lose heat to its surroundings. But if the temperature of the calorimeter remains constant for several hours it is safe to assume that it will not gain or lose any heat while remaining under the same external conditions for a few hours longer.

Before recording any measurements there is a preliminary run of about an hour. During this period the dry air is drawn through the calorimeter and the electric current adjusted to a convenient value and one that can be maintained constant. The resistances, R_1, R_2, R_3 are adjusted to give zero deflection when their respective

keys are closed. It is seldom that any readjustment is necessary, and it only remains to keep the current at the value which will maintain these balances.

The vapor-laden air leaving the calorimeter must pass through 25 centimeters of tin tubing before reaching the first H_2SO_4 tube. If its temperature is lowered the slightest, water will be deposited on the sides of the tube and thus fail to be collected and weighed. The greater part of this tube is in the water bath, but the last quarter of its length projects into the colder room. This is covered lightly with a thick cloth, under which is an incandescent lamp. This is a necessary precaution and it effectively prevents any deposition of water within the tube. At least two experiments were lost by neglecting this detail.

The Observers. — Two assistants were required whenever a set of experiments were made, and I am indebted to my students in Electrical Measurements, who not only have become deeply interested in this work, but have rendered invaluable service in its accomplishment. One man sits at the galvanometer and maintains the heating current at a constant value. This is monotonous work and he is relieved in two hours by another observer. The other assistant controls the air current, letting just enough through the calorimeter to hold the thermometer reading constant. The time of the writer was fully occupied with the collection of the water vapor, measurements of time, temperature, etc., and seeing that everything went smoothly and well. Every observation and adjustment, except those of keeping constant the air current and the electric current, fell to his lot.

Changing the Tubes. — At the close of the preliminary run and with the calorimeter at precisely its equilibrium temperature, the weighed H_2SO_4 tubes were inserted in the air current. On the even minute, as closely as could be read on the watch, the rubber tube leading to the aspirator was slipped off. The air current through the calorimeter stops entirely, and even circulation of the air is prevented by the deep water trap. There can be no question that the preliminary run is ended and that it ended when the rubber tube was disconnected, although the electric current is still flowing as before. The temporary acid tubes are quickly replaced by the two

that have just been weighed and the rubber tube connected to them, the air being cautiously started through the new tubes in order that a too sudden impact of the air current may not blow out any of the acid. The whole process is completed in a few seconds and all runs on quietly again. The temperature of the calorimeter rises quickly with the stopping of evaporation, but increases only a few hundredths of a degree before it is checked by the starting of the air current. For a few minutes the air current is increased above normal, the increased evaporation quickly reducing the temperature to normal, and then a little below for a few moments.

Thus the water vapor brought away is that evaporated by the heat of the current while the tubes were being changed as well as later, and the experiment begins, strictly, at the ending of the preceding run. Nor can there be an appreciable loss of vapor during the changing of the tubes for the volume that could diffuse out of the end of a five millimeter tube in a few seconds is insignificant when compared with the 160 liters drawn through during the remainder of the run.

Weighing the Tubes. — The tubes were never touched with the fingers but were always handled with a clean cloth. They were weighed on a sensitive balance which was provided with doors at each end of the case. The glass front was always kept closed, even the small weights being introduced through the end door. Thus there was small chance of vapor from the breath condensing on the outside of the tubes or on any part of the balance. During the few moments the tubes were on the balance the rubber caps which closed them from the outside air were removed. Very little water can get into the tube in this short time, while the rubber caps make unsatisfactory and variable weights if left on the tubes. Here again the use of the counterpoise appears. If any water should get into the tube while it is being weighed the counterpoise, being in every way like the other tube, will also gain a like amount. Thus the difference in weight between the two tubes would be unaffected.

RESULTS.

In the following tables are recorded the data obtained in twenty-two experiments, which are all that are entirely free from known

defects or accidents. The preliminary experiments are valueless as the current supply was too unsteady to give them any weight and the weighings were made without the use of a counterpoise. A complete set of experiments at 10° C. was rendered useless by an accident which destroyed the thermal balance of the calorimeter. Some other experiments are discarded because of a leak in the calorimeter by which air, and therefore vapor, from the room could enter and unduly increase the amount of water collected in the H_2SO_4 tubes.

The first table gives the observed time of beginning and ending each experiment. The duration, corrected for the rate of the watch, is expressed in mean solar seconds as measured by the standard clock. The mean reading of the thermometer in the calorimeter is

TABLE I.

Showing the Temperature at which the Water was Evaporated; also the Duration of Each Experiment.

Date.	Duration of Experiments.			Corrected Duration.	Temp. of Water in Calorimeter.	
	Beginning.	End.	Duration.		Therm.	Temp.
1907.			secs.	T.	(ave.)	
Feb. 7	11 : 00 : 00	1 : 00 : 00	7,200	7,197	4.982	21.18
"	1 : 00 : 00	3 : 00 : 00	7,200	7,197	4.988	21.19
"	3 : 00 : 00	5 : 00 : 00	7,200	7,197	4.975	21.18
Feb. 8	9 : 00 : 00	11 : 00 : 00	7,200	7,197	4.961	21.16
"	11 : 00 : 00	1 : 00 : 00	7,200	7,197	4.963	21.16
"	1 : 00 : 00	3 : 00 : 00	7,200	7,197	4.961	21.16
"	3 : 00 : 00	5 : 00 : 00	7,200	7,197	4.965	21.16
Feb. 9	11 : 00 : 00	1 : 00 : 00	7,200	7,197	5.004	21.20
"	1 : 00 : 00	3 : 00 : 00	7,200	7,196	4.997	21.20
"	3 : 00 : 00	5 : 00 : 00	7,200	7,197	5.000	21.20
Mar. 9	9 : 40 : 00	11 : 40 : 00	7,200	7,197	5.865	21.14
"	11 : 40 : 00	1 : 05 : 00	5,100	5,098	5.869	21.15
Mar. 16	9 : 20 : 00	11 : 20 : 00	7,200	7,203	4.796	13.96
"	11 : 20 : 00	1 : 40 : 00	8,400	8,404	4.788	13.95
"	1 : 40 : 00	3 : 35 : 00	6,900	6,903	4.788	13.95
"	3 : 35 : 00	5 : 40 : 00	7,500	7,504	4.790	13.95
Mar. 23	10 : 10 : 00	12 : 10 : 00	7,200	7,205	4.577	28.06
"	12 : 10 : 00	2 : 11 : 00	7,260	7,265	4.579	28.06
"	2 : 11 : 00	3 : 40 : 00	5,340	5,344	4.577	28.06
"	3 : 40 : 00	5 : 10 : 00	5,400	5,403	4.574	28.06
Apr. 6	9 : 45 : 00	11 : 45 : 00	7,200	7,202	4.324	39.80
"	11 : 45 : 00	1 : 45 : 00	7,200	7,203	4.325	39.80

TABLE II.

Giving the Amount of Water Evaporated within the Calorimeter During Each Experiment.

Date.	Tube.	Weighings.			Water Evaporated.
		Before.	After.	Gain.	
Feb. 7	E	26.1696	29.2330	3.0634	3.0634
	G	29.1052	29.1050	0000	
"	B	25.8130	28.8696	3.0566	3.0566
	C	21.5766	21.5765	0000	
"	E	29.2330	32.3020	3.0690	3.0704
	G	29.1050	29.1064	0014	
Feb. 8	B	28.8722	31.9287	3.0565	3.0565
	C	21.5769	21.5765	0000	
"	E	32.3043	35.3730	3.0687	3.0722
	G	29.1065	29.1080	0035	
"	B	31.9287	34.9888	3.0601	3.0618
	C	21.5765	21.5782	0017	
"	E	35.3730	38.4238	3.0508	3.0573
	G	29.1090	29.1155	0065	
Feb. 9	E	38.4246	41.4660	3.0414	3.0571
	G	29.1130	29.1287	0157	
"	B	38.0310	41.0634	3.0324	3.0619
	C	21.5836	21.6131	0295	
"	E	41.4660	44.5012	3.0352	3.0573
	G	29.1287	29.1508	0221	
Mar. 9	B	29.8615	32.6373	2.7758	2.7809
	C	21.6228	21.6279	0051	
"	E	34.0210	35.9190	1.8980	1.9014
	G	29.1545	29.1579	0034	
Mar. 16	B	36.0128	37.8443	1.8315	1.8567
	C	21.6298	21.6550	0252	
"	E	37.2865	39.4596	2.1731	2.1740
	G	29.1583	29.1592	0009	
"	B	37.8443	39.6200	1.7757	1.7872
	C	21.6550	21.6665	0115	
"	E	39.4596	41.4018	1.9422	1.9424
	G	29.1592	29.1594	0002	
Mar. 23	E	33.5136	36.6135	3.0999	3.1007
	G	39.5398	39.5406	0008	
"	B	27.9066	31.0345	3.1279	3.1298
	C	25.0065	25.0084	0019	

TABLE II. — *Continued.*

Giving the Amount of Water Evaporated within the Calorimeter During Each Experiment.

Date.	Tube.	Weighings.			Water Evaporated.
		Before.	After.	Gain.	
Mar. 23	E	36.6135	38.9152	2.3017	2.3017
	G	39.5406	39.5404	0000	
"	B	31.0345	33.3560	2.3215	2.3249
	C	25.0084	25.0118	0034	
Apr. 6	B	33.3567	36.5984	3.2417	3.2461
	C	25.0072	25.0116	0044	
"	E	38.9135	42.1544	3.2409	3.2528
	G	39.5372	39.5491	0119	

shown in next to the last column. The scale was divided into hundredths of a degree and easily read to thousandths. It was thus comparatively easy to keep the temperature variations within one or two hundredths of a degree. The actual temperature is given in the last column, these values being in terms of thermometer "B.S. 2107" as corrected by the certificate of the Bureau of Standards dated Dec., 1906.

Table II. gives the weighings of the H_2SO_4 tubes. As there were always two of these in series the gain in weight of both is taken as giving the total amount of water collected by them. It will be noticed that the second tube gains an appreciable amount only after the first one has caught several grams of water.

The electrical measurements are recorded in Table III. During the experiments at about 21° C. the standard cell, in its beaker of kerosene, was kept in one corner of the constant temperature bath. When the bath was changed to other temperatures the cell was placed in a large tank of water in an adjoining room where its temperature would change but slowly, and always be moderate.

The resistances, R_1, R_2, R_3, have the significance shown in Fig. 2. The 218.9 ohms under R_2 is applied as a correction because of a voltmeter current through the standard during the first three days in addition to the heating current. It required 218.9 ohms in R_2 to balance this voltmeter current alone: hence this is to be subtracted from the total of 4,000 ohms to give the resistance necessary to

TABLE III.

The Electrical Measurements, Showing the Energy Supplied by the Electric Current for the Evaporation of Water.

Date.	Standard Cell.		Box Coils—as Marked.			Calibration Corrections.			Gal. Readings.			Corrections.		Corrected Resistances.			R	$E=E_s\frac{R_3}{R_1}$	$I=\frac{E_sR_3}{RR_1}$
	Temp.	E_s	R_1	R_2	R_3	r_1	r_2	r_3	d_1	d_2	d_3	r_2'	r_3'	R_1	R_2	R_3			
						ohms.	ohms.	ohms.	mm.	mm	mm.	ohms.	ohms						
Feb. 7	21.1	1.01893	3,000	4,000	11,077	+.32	+.02	+.55	0	0	+.7	0	−3.3	3,000.3	3,781.1	11,074.2		3.7609	.27725
"	"	"	"	−218.9	"	"	"	"	0	−.3	+.1	+1.4	−.5	"	3,782.5	11,077.0		3.7618	.27735
"	"	"	"	"	"	"	"	"	0	−.6	−.1	+3.0	+.5	"	3,784.1	11,078.0		3.7621	.27747
Feb. 8	"	"	"	"	11,074	"	"	"	0	0	0	0	0	"	3,781.1	11,074.5		3.7610	.27725
"	"	"	"	"	"	"	"	"	0	+.1	−.4	−.5	+2.0	"	3,780.6	11,076.5		3.7617	.27721
"	"	"	"	"	"	"	"	"	0	−.1	−.2	+.5	+1.0	"	3,781.6	11,075.5		3.7613	.27729
"	"	"	"	"	11,074.7	"	"	"	0	+.1	−.1	−.5	+.5	"	3,780.6	11,075.7		3.7614	.27721
Feb. 9	"	"	"	"	11,075	"	"	"	0	0	−.3	0	+1.2	"	3,781.1	11,076.7		3.7617	.27725
"	"	"	"	"	"	"	"	"	0	0	0	0	0	"	3,781.1	11 075.5		3.7613	.27725
"	"	"	"	"	"	"	"	"	0	+.1	0	−.5	0	"	3,780.6	11,075.5		3.7613	.27721
Mar. 9	"	"	"	6,000	6,337.9	"	+.22	+.33	0	0	+1.7	0	−3.2	"	6,000.2	6,335.0		2.1514	.43997
"	"	"	"	"	6,113.6	"	"	+.25	0	−.1	+4.1	+.2	−7.7	"	6,000.3	6,106.2		2.0737	.43998
Mar. 16	19.2	1.01885	"	3,000	8,539.5	"	+.32	+.76	0	0	+.1	0	−.2	"	3,000.3	8,540.1		2.9000	.21998
"	20.0	1.01888	"	"	8,539	"	"	"	0	0	+.3	0	−.5	"	3,000.3	8,539.3		2.9000	.21999
"	20.8	1.01891	"	"	"	"	"	"	0	0	0	0	0	"	3,000.3	8,539.8		2.9001	.22000
"	21.6	1.01894	"	"	8,538.6	"	"	"	0	−.1	−.1	+.2	+.2	"	3,000.5	8,539.6		2.9001	.22002
Mar. 23	25.0	1.01907	4,000	5,000	14,961	+.02	−.08	+.94	0	0	0	0	0	4,000.0	4,999.9	14,961.9		3.8118	.27503
"	25.0	1.01907	"	"	"	"	"	"	0	0	+.1	0	−.5	"	"	14,961.4		3.8117	.27503
"	25.2	1.01908	"	"	"	"	"	"	0	0	0	0	0	"	"	14,961.9		3.8119	.27504
"	25.7	1.01910	"	"	"	"	"	"	0	0	+.1	0	−.5	"	"	14,961.4		3.8118	.27504
Apr. 6	20.4	1.01890	"	"	15,494	"	"	+.79	0	0	0	0	0	"	"	15,494.8		3.9468	.27499
"	21.8	1.01895	"	"	"	"	"	"	0	0	0	0	0	"	"	15,494.8		3.9470	.27500

The Standard Resistance was 4.6315 ohms in each experiment.

balance the heating current alone. In some cases the value of R_3 was changed during an experiment. The recorded value is then the time average of the different actual resistances which explains the decimal numbers in this column. The resistance coils of this box were carefully compared with each other by the Carey Foster method and the corrections thus found are shown in the columns, r_1, r_2, r_3; r_1 being the correction to be added to R_1, and similarly for the others. As the ratio of two of these resistances is always used the temperature correction does not enter.

The galvanometer readings, d_1, d_2, d_3, are the time averages of the readings observed on closing the keys, K_1, K_2, K_3, respectively. These readings are always small and as some are positive and others negative the resultant sum for a two hour period is nearly zero. If it is not zero it means that the current was not kept at quite the assigned value. The columns r_2' and r_3' show the amounts by which R_2 and R_3 should have been changed to exactly balance the actual current which went to the heating coil. Both sets of corrections are applied to give the "corrected resistances." The temperature of the standard resistance, R, was read and recorded for each experiment. The lowest was 15° C. on March 16, and the highest was 25° C. on April 6. Its resistance was 4.6315 ohms. with no change in these five figures for this range of temperature,

The last column gives the current, I, through the heating coil within the calorimeter, the values being computed by the formula at the top of the column. The fall of potential, E, over the same coil is given in the preceding column. The values of E are expressed in terms of 1.43400 volts for the E.M.F. of a Clark cell at 15° C. The unit of current here used is such that 1.43400 amperes flowing through one international ohm will produce a potential difference equal to the E.M.F. of a Clark cell at 15° C.

The final results are collected in Table IV. which is largely self-explanatory. The last column gives the heat of evaporation computed by the formula at the top of the column. These values are in sets corresponding to the four different temperatures at which they were determined. The large number of determinations at 21° C. were made partly because the value of L at this temperature was most desired, but largely to test the reliability and accuracy of the

method. Each day's work stands independently, the apparatus being dismantled and the calorimeter refilled with a fresh supply of distilled water for each set of determinations.

Although only two values were obtained on March, yet these are especially valuable as the calorimeter used on that date was one of the earliest forms. These determinations are therefore free from any constant bias due to the particular form of calorimeter used and which might affect all the others. It will be noticed, however, that these two results agree very closely with the others obtained at the same temperature.

TABLE IV.

Collected Data. Giving the Results Obtained in Each Experiment.

Date.	Temperature of Water.	Water Evaporated.	Duration.	E	I	$L=\frac{EIT}{W}$
		grams.	secs.	volts.	amperes.	joules.
Feb. 7	21.18	3.0634	7,197	3.7609	.27725	2,449.7
"	21.19	3.0566	7,197	3.7618	.27735	2,456.6
"	21.18	3.0704	7,197	3.7621	.27747	2,446.8
Feb. 8	21.16	3.0565	7,197	3.7610	.27725	2,455.3
"	21.16	3.0722	7,197	3.7617	.27721	2,442.8
"	21.16	3.0618	7,197	3.7613	.27729	2,451.6
"	21.16	3.0573	7,197	3.7614	.27721	2,454.5
Feb. 9	21.20	3.0571	7,197	3.7617	.27725	2,455.2
"	21.20	3.0619	7,197	3.7613	.27725	2,451.2
"	21.20	3.0573	7,197	3.7613	.27721	2,454.5
Mar. 9	21.14	2.7809	7,197	2.1514	.43997	2,449.7
"	21.15	1.9014	5,098	2.0737	.43998	2,446.3
Mar. 16	13.96	1.8567	7,203	2.9000	.21998	2,474.9
"	13.95	2.1740	8,404	2.9000	.21999	2,466.2
"	13.95	1.7872	6,903	2.9001	.22000	2,464.3
"	13.95	1.9424	7,504	2.9001	.22002	2,465.1
Mar. 23	28.06	3.1007	7,205	3.8118	.27503	2,436.0
"	28.06	3.1298	7,265	3.8117	.27503	2,433.4
"	28.06	2.3017	5,344	3.8119	.27504	2,434.1
"	28.06	2.3249	5,403	3.8118	.27504	2,436.4
Apr. 6	39.80	3.2461	7,202	3.9468	.27499	2,408.0
"	39.80	3.2528	7,203	3.9470	.27500	2,403.6

The mean value of L for each temperature is,

Temperature.	Heat of Evaporation.
13.95	2,467.6
21.17	2,451.2
28.06	2,435.0
39.80	2,405.8

These values are plotted as ordinates of the lower curve in Fig. 3, which is the straight line that most nearly represents the points. A slightly convex curve would fit the observations better and might possibly be nearer the truth. But in view of the uncertainty regarding the location of such a curve at 100° C. it is deemed best to merely draw the nearest straight line, as this represents the results

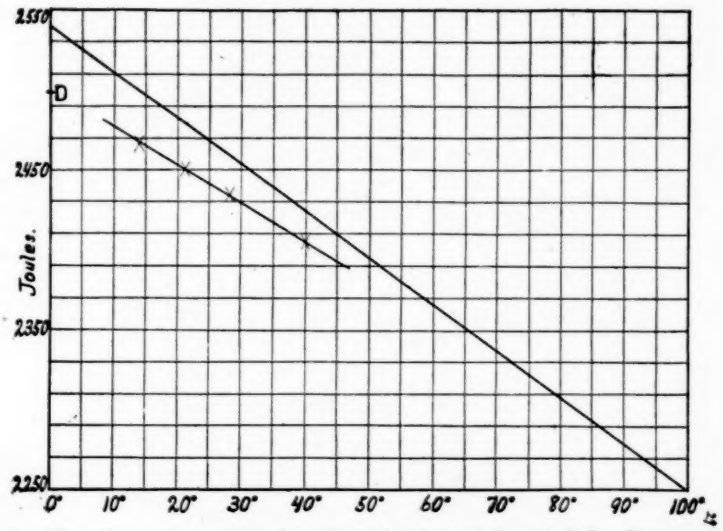

Fig. 3. The short curve gives the values of L for 10° C. to 40° C. For comparison Regnault's formula is plotted from 0° C. to 100° C. Dieterici's value is shown at D.

sufficiently well over the range of temperature which has been investigated.

The individual determinations are too close to each other to appear as separate points and only the mean value for each temperture is shown. In computing the probable error of the curve however, each one of the twenty-two experiments is given an equal weight, the line being drawn to make this a minimum. The equation of this line is

$$L = 2{,}502.5 - 2.43T$$

where T is the temperature Centigrade and L is given in international joules.

Other Investigations.

Dieterici. — Undoubtedly the best determination previously made of the heat of evaporation of water at low temperatures is that of

Dieterici. In 1889 he evaporated water within a Bunsen ice calorimeter. The water was placed in a small bulb within the inner tube of the calorimeter. When thermal equilibrium was attained the water was all evaporated by reduced pressure, the heat required being furnished by the further freezing of the ice mantle. Assuming that one mean calorie expelled 0.01544 gram of mercury, he obtained[1] the value 596.80 mean calories for the heat of evaporation at 0° C.

Using the best modern apparatus and methods he has recently determined[2] that 0.015491 gram of mercury corresponds to one mean calorie. In a later investigation with the Bunsen calorimeter and in which the heat was generated by an electric current, Dieterici finds[3] that it requires 4.1925 joules to draw 0.015491 gram of mercury into the calorimeter. The electrical units are expressed in terms of a Weston element, and probably the result is given in Reichsanstalt joules. Since Reichsanstalt volts are larger than international volts by the factor 1.00081, and this factor enters twice in the expression for joules, we have for the heat of evaporation of water at 0° C.,

$$L = 596.80 \times \frac{.015440}{.015491} \times 4.1925 \times (1.00081)^2$$

$$= 2497.9 \text{ international joules,}$$

that is, when expressed in terms of 1.43400 volts for a Clark cell at 15° C. This value is shown at the point D, Fig. 3.

Regnault. — Regnault's familiar equation, $L = 606.5 - 0.695T$, exterpolated largely from results between 63° C. and 230°[4] C. is known to give much too large values at low temperatures, and even Regnault recognized that his determinations at low temperatures were not as reliable as those at the higher temperatures. Yet, strange as it may seem, these values are quoted to-day more often than the results of all other determinations combined. Partly for this reason, the above equation is plotted in Fig. 3, and it shows clearly how much larger values it gives than those which have since

[1] Ann. der Phys., Vol. 37, p. 504, 1889.
[2] Ann. der Phys., Vol. 16, p. 603, 1905.
[3] Ann. der Phys., Vol. 16, p. 619, 1905.
[4] Institute de France, Mem. Acad. Sci., Vol. 21, pp. 1–748, 1847.

been determined. The individual determinations vary considerably among themselves and cannot be accorded much weight.

Some years later Regnault made two series of experiments[1] in which dry air was drawn through the water. The results were not very satisfactory. The mean of the first series agreed quite closely with his formula, but the second series gave results two per cent. above the formula. Regnault accepted the result with the remark that this descrepancy should be looked into, but he recorded no further experiments along this line. In fact his apparatus was ill-fitted for such work, as the corrections amounted to a large fraction of the total measurements. He accepted these results, however, as demonstrating that the heat required to slowly evaporate water into air at atmospheric pressure is the same as when it boils rapidly under reduced pressure.

Griffiths. — Some years ago Griffiths made an elaborate investigation[2] into the heat of evaporation of water. His original intention was to cover the range from 10° C. to 60° C., but only a few results at 30° C. and at 40° C. are given, and he says "had time permitted I should have performed more experiments especially at 30° C." His preliminary method was to draw dry air through the water, but this failed to give concordant results and the final determinations were made by the more common method of reduced pressure.

A weighed amount of water was placed in a glass tube within the calorimeter and flowed out through a fine opening as fast as it was evaporated. Heat was supplied by an electric current which was stopped, as nearly as possible, when the last of the water was evaporated. The experiments were conducted at a constant temperature and appear to have been very carefully performed. The heat furnished by the current was computed from the formula $H = E^2T/R$, the value of E being measured in terms of a Clark cell the E.M.F. of which was taken as 1.4342 volts at 15° C. He expressed his results in calories, using 4.199 for the mechanical equivalent of heat. Using the same factor to translate the results back into joules gives

2429.3 joules at 30°.00 C.
and 2403.6 joules at 40°.15 C.

[1] Institute de France, Mem. Acad. Sci., Vol. 26, pp. 883–906, 1862.
[2] Phil. Trans., Vol. 186 A, pp. 261–342, 1895.

when expressed in terms of 1.43400 volts for the Clark cell at 15° C.

Probably these determinations at 30° C. and at 40° C. are reliable; but it is questionable whether his formula, $L = 596.73 - .601\ T$, should be exterpolated to 0° C. and 100° C. To be sure it thus agrees at 100° C. with Regnault's published value, but there is reason to believe that this is too low. At 0° C. the formula, as expressed in calories, is identical with Dieterici's results as first published. As now corrected the latter values are lower by 1 part in 300, thus leaving the formula without experimental basis below 25° C.

Henning. — During the progress of this work there has appeared [1] the account of a similar investigation for the range from 50° C. to 100° C. A few determinations were made at 30° C. but for some reason are given only one eighth the weighting accorded the determinations at each of the higher temperatures. The water was made to boil under reduced pressure, the vapor being condensed and weighed. Heat was supplied by an electric current and measured in terms of a Weston standard cell. Probably the results were obtained in terms of Reichsanstalt volts. They are expressed in 15-degree calories by means of the factor 4.188 joules per calorie. I have reduced them back to international joules by multiplying by this same factor and also by 1.0016.

Results of Other Investigations.

	Temperature.	Heat of Evaporation.	
		As Reported.	As Computed.
		Calories.	International Joules
Dieterici.	0	596.80	2,497.9
Griffiths.	30.00	578.70	2,429.3
"	40.15	572.60	2,403.6
Henning.	30.12	579.0	2,428.7
	49.14	569.55	2,389.0
	64.85	559.47	2,346.8
	77.34	552.47	2,317.4
	89.29	545.76	2,289.3
	100.59	538.25	2,257.8

Whether expressed in calories or joules, most of the results appear rather high. This is very apparent at 100° C. where the

[1] Ann. der Phys., Vol., 21, pp. 849–878, 1906.

familiar number 537 is exceeded by nearly two units. However Henning is not alone in finding this larger value, and it may be that the accepted value is too low. Certain it is that this important constant should be redetermined with modern appliances and with a greater degree of precision than has yet been done.

In the preceeding table are collected the results mentioned above:

Summary.

1. Water is evaporated by passing through it a stream of dry air. As now developed, this method has proved entirely satisfactory and can be used for any temperature at which the surrounding water bath can be maintained constant. The usual method of boiling water under reduced pressure is not suitable for low temperatures as it is difficult to maintain constant and steady evaporation. This explains why previous investigators have confined their work to the higher temperatures. With the present method the temperature of the evaporating water remains constant, and all thermal corrections are eliminated.

2. The heat of evaporation of water at temperatures between 14° C. and 40° C. is given by the formula

$$L(\text{in joules}) = 2502.5 - 2.43T.$$

The "probable error" of values computed from this formula is 0.5 joule.

3. These results are expressed in international joules: that is, in terms of the international ohm and 1.43400 volts for the E.M.F. of the Clark cell at 15° C. They are thus given in terms of a definite and reproducible unit. Should the E.M.F. of the Clark cell be found to be less than this value by 1 part in 1,000, all of the results here reported will be decreased by 2 parts in 1,000. They will then be expressed in terms of the new unit as exactly as they are now given in international joules.

4. It is interesting to note that the results obtained by this method are in line with those of Dieterici at 0° C. and those of Griffiths at higher temperatures, although obtained by totally different methods. It might be inferred, *a priori*, that it would require the same amount of heat to evaporate a gram of water whether the

external pressure be great or small. Nevertheless it is a source of satisfaction to have this experimental demonstration that the heat of evaporation is independent of the pressure.

5. The value of the heat of evaporation at five degree intervals is given in the following table. For comparison the results are also given in mean calories, assuming that a mean calorie is equivalent to 4.1887 international joules.[1]

This makes the formula read

$$L \text{ (in mean calories)} = 597.44 - .580\ T.$$

Heat of Evaporation of Water.

Temperature Centigrade.	International Joules.	Mean Calories.	Temperature Centigrade.	International Joules.	Mean Calories.
10	2,478.2	591.64	30	2,429.6	580.04
15	2,466.0	588.73	35	2,417.4	577.12
20	2,453.9	585.84	40	2,405.3	574.24
25	2,441.7	582.93	45	2,393.1	571.33

UNIVERSITY OF MICHIGAN,
June 1, 1907.

[1] Phil. Trans., Vol. 199 A, p. 149, 1902.

SPECIFIC HEAT OF SOLUTIONS. V.

By William Francis Magie.

1. This paper is a continuation of others, published under the same title in previous volumes of this Review. It deals with the heat capacities of electrolytic solutions and the possibility of representing them by a simple formula involving the dissociation factor. Most of the measurements of heat capacity employed are those of Julius Thomsen.[1] The measurements of solutions of potassium chloride were made by myself, and those of sodium and potassium nitrate, barium and strontium chloride, and barium and strontium nitrate by Mr. R. E. Trone, Instructor in physics at Princeton University. The calorimeter used by us was the modified form of the Pfaundler calorimeter described in No. III. of this series. It gives me pleasure to have an opportunity to express my obligations to Mr. Trone for his faithful and accurate work.

The volumes of electrolytic solutions exhibit in general the same peculiarities as their heat capacities, and admit of representation by a similar formula. I have accordingly introduced a discussion of these volumes *pari passu* with that of the heat capacities. Most of the measurements of volume employed were made by Thomsen.[2] Those for strontium chloride were worked over from data given by Kohlrausch and Holborn.[3] Mr. Trone, using a standardized Mohr's balance, contributed those for barium chloride and for barium and strontium nitrate.

2. *Peculiarity of the Heat Capacities of Electrolytes.* — The solutions which Thomsen studied were made up by dissolving one gram-molecule of the solute in some known number, N, of gram-molecules of water. The heat capacity of such a solution was determined and the heat capacity of the N gram-molecules of water subtracted

[1] Thermodynamische Untersuchungen, Vol. I., Pogg. Ann., 142, p. 335.

[2] L. c.

[3] Leitvermögen der Elektrolyte, p. 145.

from it. The difference gave the apparent heat capacity of one gram-molecule of the solute — or its apparent molecular heat — in a solution of that concentration. It was found that the apparent molecular heat of the solute generally diminishes with increasing dilution, so that in practically all cases it becomes negative long before the solution becomes very dilute.

Similarly, the apparent molecular volumes of the same solutes diminish with increasing dilution, though it is only in a very few cases that they become negative within the range of Thomsen's observations.

3. *General Conclusion.* — These results demonstrate that in solutions, at least in electrolytic solutions, the water is affected by the presence of the solute in such a way that its heat capacity and specific volume are diminished.[1] They are therefore conclusive in favor of some form of what we may call the association theory of solutions, in distinction from that theory of solutions in which the solute is considered as having the properties of a gas, and the solvent as being merely an inert medium in which the molecules or ions of the solute are suspended. As will be seen later, they indicate that the association between the molecules or ions of the solute and the water, within the range of concentration covered by the observations, is fairly definite in its effects, so that we may conceive of each molecule or ion of solute as surrounded by a group of a definite number of water molecules, which are within its sphere of influence and affected by it. There is little or no evidence that the numbers of water molecules in these groups change with the concentration. In this respect the hypothesis which will explain the results under consideration differs from that adopted by Jones and his fellow-workers [2] to explain the abnormalities of the freezing and boiling points observed by them in concentrated solutions.

The conclusion that the negative values of the apparent molecular heats prove that the heat capacity of the water is affected by the presence of the solute has already been drawn by Tammann.[3]

[1] Magie, Bull. Am. Phys. Soc., Vol. 2, no. 2; this Review, XIII., p. 91; XVIII., p. 449.

[2] Am. Chem. Jour., 23, p. 89; Zeitschrift für Phys. Chem., XLVI. p. 244; XLIX., p. 385; this Review, XVIII., p. 146.

[3] Zeitschrift für Phys. Chem., XVIII., p. 625.

4. *Formulas to Represent Heat Capacities and Volumes of Solutions.*—An examination of the tables which follow will show that the heat capacities of the solutions of any one solute, made up by dissolving one gram molecule of the solute in N gram-molecules of water, may be represented by the formula

$$H = WN + A + Bp.$$

In this formula W is the molecular heat of water, or 18, A and B are constants, and p is the dissociation factor, obtained in the usual way as the ratio of the molecular conductivity of the solute in a solution of the given concentration to the molecular conductivity of the same solute in an infinitely dilute solution. The values of p are calculated from the molecular conductivities given in the tables of Kohlrausch and Holborn, with the help of their tables of specific gravities.

A similar formula,

$$V = UN + D + Ep,$$

will represent the volumes of the various solutions. In this formula U is the molecular volume of water, D and E are constants, and p again the dissociation factor.

NaCl. $A = 39$, $B = -70$; $D = 25.86$, $E = -12$

N	p	H Obs. *Th.*	H Calc.	V Obs. *Th.*	V Calc.
10	0.42			200.9	200.8
20	.544	361	361.0	379.3	379.3
30	.613	536	536.1	558.4	558.5
50	.668	892	892.2	917.8	917.8
100	.729	1,788	1,788.0	1,816.1†	1,817.1
200	.780	3,578†	3,584.4	3,616.0	3,616.4

KCl. $A = 96$, $B = -149$; $D = 47$, $E = -25$.

N	p	H Obs. *M.*	H Calc.	V Obs. *Th.*	V Calc.
15	0.672			300.4	300.2
30	.728			569.0	568.8
50	.753	880.7	882.0	928.2	928.2
100	.787	1,777.6	1,777.2	1,827.3	1,827.3
200	.818	3,571.8	3,572.2	3,625.0	3,626.0
300	.835	5,370.2	5,370.2		
400	.848	7,167.1	7,168.1		

[1] The letters *Th.*, *M.*, *Tr.*, *K*, prefixed to the tabulated observations, indicate the various observers, Thomsen, Magie, Trone, Kohlrausch.

NH_4Cl. $A = 39$, $B = -62.1$; $D = 43.5$, $E = -7.5$.

N	p	H Obs. Th.	H Calc.	V Obs. Th.	V Calc.
10	0.593	181.6	182	219.0	219.1
25	.703	443.6	445	488.2	488.2
50	.742	893	893	937.8	937.9
100	.777	1,791	1,791	1,837.7	1,837.7
200	.806	3,588	3,589	3,637.6	3,637.5

HCl. $A = 4$, $B = -41$; $D = 27$, $E = -10.56$.

N	p	H Obs. Th.	H Calc.	V Obs. Th	V Calc.
20	0.596	339	339.6	379.2	380.7
50	.773	873	872.4	918.8	918.8
100	.84	1,770	1,769.6	1,818.5	1,818.1
200	.877	3,561†	3,568	3,617.7	3,617.7

$NaOH$. $A = 32$, $B = -65.6$; $D = 7$, $E = -15$.

N	p	H Obs. Th.	H Calc.	V Obs. Th	V Calc.
30	0.602	533	533	537.9	538
50	.696	885	886	896.4	897
100	.775	1,781	1,781	1,795.9	1,795.4
200	.824	3,578	3,578	3,594.8	3,594.7

KOH. $A = 59$, $B = -110$; $D = 22$, $E = -21$.

N	p	H Obs. Th.	H Calc.	V Obs. Th.	V Calc.
30	0.69	522	523	547.5	547.6
50	.76	876	875	906.3	906.1
100	.82	1,770	1,769	1,804.9	1,804.8
200	.855	3,565	3,565	3,604.1	3,604.1

$\frac{1}{2}H_2SO_4$. $A = 27.5$, $B = -40$; $D = 22$, $E = -10$.

N	p	H Obs. Th.	H Calc.	V Obs. Th.	V Calc.
9.5	0.337	180.6	185	189.8	189.6
24.5	.463	450	450	458.3	458.4
49.5	.510	898.5	898.1	907.7	907.9
99.5	.532	1.797.5	1.797.5	1,807.7	1,807.7

$NaNO_3$. $A = 55$, $B = -75$; $D = 37.5$, $E = -13.5$.

N	p	H Obs. Tr.	H Calc.	V Obs. Th	V Calc.
10	0.34			212.5	213
25	.514			480.4	480.7
50	.618	908.2	908.6	929.2	929.2
100	.698	1,802.7	1,802.7	1,828.2	1,828.2
200	.757	3,598	3,598.2	3,627.0	3,627.5
300	.785	5,396	5,396.1		

KNO_3. $A = 78$, $B = -120$; $D = 47.5$, $E = -12$.

N	p	H Obs. *Tr.*	H Calc.	V Obs. *Th.*	V Calc.
25	0.55	458.6	462	490.7	490.9
50	.633	902.2	902.1	939.8	939.8
100	.705	1,793.6	1,793.4	1,839.2	1,839.1
200	.76	3,586.6	3,586.8	3,638.3	3,638.4
300	.79	5,383.8	5,383.2		
400	.81	7,182.0	7,180.8		

$\frac{1}{2}BaCl_2$. $A = 10$, $B = -65$; $D = 29.2$, $E = -25$.

N	p	H Obs. *Tr.*	H Calc.	V Obs. *Tr.*	V Calc.
50	0.548	875.5	874.4	915.2	915.5
100	.606	1,770.5	1,770.6	1,814.2	1,814.1
200	.662	3,567	3,566.9	3,612.8	3,612.7
300	.69	5,365.6	5,365.2	5,411.8	5,412.0

$\frac{1}{2}SrCl_2$. $A = 16.6$, $B = -76$; $D = 18.8$, $E = -10$.

N	p	H Obs. *Tr.*	H Calc.	V Obs. *K*	V Calc.
50	0.56	873.5	874	912.6	913.2
100	.62	1,770.3	1,769.6	1,812.6	1,812.6
150	.66	2,666.3	2,666.6	2,712.3	2,712.2
200	.681	3,565.4	3,564.9	3,612.0	3,612.0
300	.717	5,361.5	5,362.1	5,411.6	5,411.6

$\frac{1}{2}Ba(NO_3)_2$ $A = 31$, $B = -76$; $D = 25.3$, $E = 0$.

N	p	H Obs. *Tr.*	H Calc.	V Obs. *Tr.*	V Calc.
100	0.469	1,795.7	1,795.4	1,825.5	1,825.3
150	.526			2,725.3	2,725.3
200	.563	3,588.4	3,588.2	3,625.3	3,625.3
250	.593			4,525.3	4,525.3
300	.618	5,382.3	5,384.0	5,426.3	5,425.3

$\frac{1}{2}Sr(NO_3)_2$. $A = 9$, $B = -40$; $D = 24.8$, $E = -3$.

N	p	H Obs. *Tr.*	H Calc.	V Obs. *Tr*	V Calc.
50	0.442	895	892	923.5	923.5
100	.533	1,787.8	1,788	1,823.2	1,823.2
200	.607	3,584.8	3,584.7	3,623.0	3,623.0
300	.648	5,383.5	5,383.1		

The dissociations used here were determined by Professor William Foster (this REVIEW, VIII., p. 257). The curve of dissociations, plotted from Kohlrausch's data, is suspiciously irregular.

A few of the observations in these tables are marked with a (†). They differ by more than the probable error from the calculated numbers. In every one of these cases, if the curve of specific heats or of specific gravities is plotted, it will be found that the discordant result corresponds to an aberrant part of the curve; and when the curve is smoothed, the heat capacity or the volume calculated from the smoothed curve is in good accord with the quantity calculated from the formula. I think there can be no doubt that these discrepancies are due to experimental errors, and that it is legitimate to remove them by smoothing the curves. An error of two tenths of one per cent. in the determination of the specific heat would account for the largest discrepancy found.

6. *Deduction of Formulas.* — The formulas given in § 4, from which the results of the tables have been calculated, admit of a simple interpretation. Let us suppose that n gram-molecules of the solute are dissolved in N gram-molecules of water, and that p gram-molecules of solute are dissociated into ions. Let us suppose further that each actual molecule and each ion of the solute collects around itself a group of water molecules and in general affects and modifies the heat capacity of the group. The heat capacity of the solution is additive of the heat capacities of the several homogeneous parts of the solution, and on our suppositions is the sum of the heat capacities (1) of the water outside the groups of water molecules affected by the solute; (2) of the undissociated molecules and the water affected by them; (3) of the ions and the water affected by them. This sum may be written out by the help of the following symbols:

M, S, N, molecular weight, specific heat, number of gram-molecules of water.

m, n, molecular weight, number of gram-molecules of solute.

p, number of gram-molecules dissociated into ions.

a, α, number of water molecules affected by one undissociated gram-molecule of solute, and by one dissociated gram-molecule of solute, respectively.

s, σ, specific heats of the groups containing an undissociated molecule of solute and a dissociated molecule of solute, respectively.

The heat capacity of the solution may be written,

$$H = S\,(MN - a(n-p)M - apM) + s(m+aM)(n-p) + \sigma(m+aM)p$$
$$= SMN + (sm + saM - SaM)(n-p) + (\sigma m + \sigma aM - SaM)p$$
$$= WN + A(n-p) + Cp,$$

in which WN is the heat capacity of the water used in making up the solution; A is a constant depending upon the undissociated molecules and the way they affect the water; C is a constant depending in a similar way upon the dissociated molecules, or the ions formed from them, and the way they affect the water.

In case n equals 1, p is the dissociation factor, and the formula then becomes

$$H = UN + A + Bp,$$

in which $B = C - A$.

On the same suppositions as to the constitution of a solution, its volume may be considered as the sum of the volumes of those parts of the solution whose heat capacities have been added to express the heat capacity of the solution. Writing out in a similar way the sum of these volumes, we find that the volume of the solution is expressed by a formula of the same form as that obtained to express the heat capacity. In this formula, which may be written in the form

$$V = UN + D + Ep,$$

UN is the volume of the water used in making up the solution; D is a constant analogous to A; E is a constant, equal to $F - A$, and the constant F is analogous to C.

It thus appears that the simple suppositions which we have made as to the constitution of electrolytic solutions lead to the formulas for the heat capacity and the volume of solutions which have been shown to represent the results of observation.

7. *Formulas of Mathias and of Tammann.* — Of the formulas which have heretofore been proposed to represent the heat capacities or the specific heats of solutions, it is worth while to notice those of Mathias and of Tammann.

Mathias[1] proposed a formula for the specific heats, with two

[1] Journal de Physique, Vol. 8, p. 204.

constants, a and b, of the form

$$\gamma_n = \frac{a+n}{b+n} c;$$

in which γ_n is the specific heat of the solution containing n gram equivalents of the solvent, and c is the specific heat of the solvent. This formula sometimes represents the specific heats given by Thomsen and by Marignac exceeding well. In other cases it is not so successful. In his notice in the Comptes Rendus, 107, p. 524, Mathias presents his results for sodium chloride and hydrochloric acid solutions as examples of the applicability of his formula. In the following table his results are compared with Thomsen's observations and with the values given by the formula presented in this paper.

N	NaCl			HCl		
	H Obs. *Th.*	H Calc. *Mat*	H Calc. M	H Obs. *Th.*	H Calc. *Mat*	H Calc. M
20	361	359	361	339	338.2	339.6
30	536	534.5	536.1			
50	892	890.6	892.2	873	873.8	872.4
100	1,788	1,786.2	1,788	1,770	1,772	1,769.6
200	3,585[1]	3,584	3,584.4	3,567	3,571	3,568

These tables show that the formula of Mathias, in which the concentration, as represents by n, is taken as the variable upon which the change of specific heat depends, does not represent the observations so exactly as the formula in which the dissociation is the variable.

Tammann[2] considers the peculiarities of the heat capacities of solutions to result from the change in the heat capacity of the solvent, due to an internal pressure caused by the presence of the solute. His main principle is like that upon which the formula presented in this paper is based, except that he considers the internal pressure as affecting the whole mass of solution. As might be expected from the difficulty of obtaining the data which he needs for his computations, Tammann's calculated heat capacities do not represent the observations with any accuracy.

[1] Corrected by smoothing the curves of specific heat.

[2] Zeitschrift für Phys. Chem., Vol. 18, p. 625.

8. *Formulas for Non-electrolytic Solutions.* — In the case of solutions which are not electrolytes, in which, therefore, p is equal to zero or is negligibly small, the formula for the heat capacities reduces to the sum of two terms, of which the first represents the heat capacity of the water and the second is a constant characteristic of the solute. Such a formula has been shown [1] to represent the heat capacity of many non-electrolytic solutions. In cases in which it does not do so, as in the case of the solutions of the alcohols in water, other considerations, notably the heat developed on solution, lead us to conclude that an interaction takes place between the solvent and the solute of a nature similar in general to that supposed to occur in the cases of the electrolytes. The laws of this interaction are not known, and I have not been able to find a formula which will adequately represent the results of observation in these exceptional cases.

In the normal cases, in which the simple additive formula for the heat capacity applies, a similar formula holds for the volumes of the solutions. This has been shown by Wade and by Wanklyn [2] for solutions of cane sugar, and calculations from the data given by various observers for the volumes of solutions of dextrose, of levulose, of urea, and of glycerine, chosen as typical examples, show that the volumes of solutions of these substances obey the same law. In the tables given by Traube [3] several other examples of the same law appear.

9. *Apparent Heat Capacities and Volumes of Ions.* — The range of observation within which our formulas, especially the one for the heat capacity, can be tested, is not very great, and it is desirable to have further confirmation of our hypothesis than that afforded by the agreement between calculated and observed results shown in the tables. This may be obtained from the following considerations:

In the construction of the general formula to which both the heat capacities and the volumes conform, the ions formed by dissociation were considered together, the symbols σ and α being in a way composite quantities, expressing the action of all the ions. It is, how-

[1] Magie, this REVIEW, IX., p. 65; XIII., p. 91.
[2] Phil. Mag., June–Dec., 1891, p. 475.
[3] Liebig's Ann., 290, p. 43.

ever, possible, by introducing separate symbols for the ion of each sort, to write out the term C as the sum of similarly formed terms referring to each ion. Indicating these terms by subscripts 0 and 1 in the case of a simple binary electrolyte, like sodium chloride, C becomes

$$\sigma_1 m_1 + \sigma_1 \alpha_1 M - S\alpha_1 M + \sigma_0 m_0 + \sigma_0 a_0 M - S a_0 M.$$

If now we have another binary electrolyte, like potassium chloride, containing the ion indicated by the subscript 0 and another indicated by a subscript 2, the C for that substance becomes

$$\sigma_2 m_2 + \sigma_2 a_2 M - S a_2 M + \sigma_0 m_0 + \sigma_0 a_0 M - S a_0 M.$$

The difference between these quantities is independent of the common ion, and we should therefore expect to find their difference the same, say for the sodium and potassium compounds, whatever the common ion may be.

A similar relation may be expected to hold among the values of F relating to the volumes of the solutions.

This expectation is fairly well met in the cases of the simpler solutions, in which we have no reason to suspect that there is more than one type of dissociation. For the sodium and potassium compounds we have the following values of C and F, and of their respective differences:

Solution.	C	Diff.	F	Diff.
NaCl	−31	22	13.9	−8.1
KCl	−53		22	
NaOH	−33.6	17.4	−8	−9
KOH	−51		1	
$NaNO_3$	−20	22	24	−11.5
KNO_3	−42		35.5	

Similarly the difference of the constants C referring to the chlorine and NO_3 ions is the same for either sodium or potassium chloride and nitrate and for barium chloride and nitrate.

Solution.	C	Diff.	Solution.	Diff.	C
NaCl	−31	−11	$\frac{1}{2}BaCl_2$	−55	−10
$NaNO_3$	−20		$\frac{1}{2}Ba(NO_3)_2$	−45	

A much larger collection of data than I have been able to obtain is needed to prove that these differences are generally constant, and the matter is so complicated by the probability that more than one type of dissociation occurs in most of the available solutions, that exceptions to the rule are rather to be expected than not. Such complicated dissociations occur with the barium and strontium compounds and may account for the failure of their solutions to follow the rule to which the simpler solutions of the sodium and potassium compounds conform.

Similar differences, and interpreted in a similar manner as the differences of the effective volumes in solution of two different ions, have been discovered and fully discussed by Traube.[1]

10. *Apparent Change of the Heat Capacity of the Water Associated with the Ions.* — By proceeding upon our hypothesis a relation can be deduced from the constant C for the different solutes examined, which may serve as an additional confirmation of the hypothesis. This constant represents the heat capacity of a gram-molecule of the dissociated solute and of the water associated with it, diminished by the heat capacity of the water when not in association with the ions. If we suppose that the ions of the solute retain the heat capacities which they have as atoms, which is practically the same thing as giving to the dissociated molecule its heat capacity when in the solid state — or, if this is not known, making up a heat capacity for it by adding the atomic heats of its atoms as determined by Kopp — we then find the amount by which the heat capacity of the water associated with the dissociated molecule is diminished by subtracting its heat capacity from the value of C in each case. When this is done, the numbers obtained, with one or two exceptions, fall into two groups, which are presented in the following tables. The

	C	h	$C-h$
NaCl	−31	12.5	−43.5
NH_4Cl	−23.1	20.9	−44
$NaNO_3$	−20	23.6	−43.6
NaOH	−33.6	13.3	−46.9
HCl	−37	8.8	−45.8
		Mean	−44.8

	C	h	$C-h$
KCl	−53	12.8	−65.8
KNO_3	−42	24.1	−66.1
KOH	−51	13.3	−64.3
$\frac{1}{2}BaCl_2$	−55	9.4	−64.4
$\frac{1}{2}SrCl_2$	−59.4	9.5	−68.9
$\frac{1}{2}Ba(NO_3)_2$	−45	19.9	−64.9
		Mean	−65.7

[1] Zeitschrift für anorg. Chem., 3, p. 11.

symbol h represents the heat capacity of the molecule of solute. The values for $\frac{1}{2}H_2SO_4$ and for $\frac{1}{2}Sr(NO_3)_2$ do not fall into either of these groups. In the case of the sulphuric acid, this can be accounted for by the mode of dissociation.

Viewed in the light of our hypothesis, these results indicate that the influence of ions of various sorts upon the water molecules which surround them, though not always the same, is practically the same in many cases. It is noticeable that the sodium compounds are all in one group, the potassium compounds in the other.

If we assume that each water molecule is equally affected by the ion with which it is connected, we may use these numbers to obtain an inferior limit to the number of water molecules thus affected. The mean loss of heat capacity of the water connected with the ions of the first group is 45, that is 2.5 times the heat capacity of one water molecule. There must therefore be at least three molecules of water affected by the two ions, and that there may be so few, it must be admitted that they lose five sixths their heat capacity. So great a change in heat capacity seems to be out of the question, and one is naturally led to assume that the sphere of influence of each ion contains several water molecules. This assumption, furthermore, is supported by the occasional failure of the formulas to represent the observations for the higher concentrations. The differences between the calculated and observed heat capacities and volumes for the most dilute solutions can be ascribed to experimental errors in determining specific heats and specific gravities; but experimental errors have less influence in the case of the more concentrated solutions, and the differences between the calculated and observed results for those concentrations are no doubt real. They may easily be explained if we consider the spheres of influence of the ions, and of the undissociated molecule, as extending so far as to include groups of water molecules. When the concentration becomes such that these groups interpenetrate, or even come into frequent collision, it may reasonably be expected that their heat capacities and volumes should be altered.

11. *Nature of the Association.* — The association between the water and the solute must be of such a sort as to permit an explanation of the laws of osmotic pressure and of electrolytic conduc-

tion. It does not seem possible to account for these laws, especially for those of conduction, if the association is of such a sort as to form definite hydrates or permanent groups of water molecules around the molecules and ions of the solute. If, however, the groups are not permanent, but are loose and unstable, so that the molecules and ions of the solute can slip along through the water without dragging all the water molecules of the group with them, the way to a kinetic explanation of osmotic pressure is left open, and the different ions may still exhibit their different characteristic velocities, postulated for them in Kohlrausch's theory of electrolytic conduction.

PRINCETON UNIVERSITY,
PRINCETON, NEW JERSEY.

INDUCTANCE OF STRAIGHT CONDUCTOR.

BY K. OGURA AND C. P. STEINMETZ.

I.

THE inductance of a length l of a straight conductor is usually given by the equation

$$L = 2l \ lg\frac{D}{r} \times 10^{-9}. \qquad (1)$$

Where D = distance of return conductor, r = radius of the conductor and the total length of the conductor is assumed as infinitely great compared with l and D. This is approximately the case with the conductors of a long distance transmission line.

For infinite distance D of the return conductor, that is, a conductor without return conductor, equation (1) gives $L = \infty$, that is, a finite length of an infinitely long conductor without return condnctor has an infinite inductance L and so inversely zero capacity C.

In equation (1), the magnetic field is assumed as instantaneous, that is, the velocity of propagation of the magnetic field is neglected. With alternating currents traversing the conductor, this is permissible, when the distance to the return conductor is a negligible fraction of the wave-length. That is, if D is negligible compared with v/N, where v = velocity of light, N = frequency of alternating current. It obviously is not permissible in a conductor having no return conductor.

If a conductor conveying an alternating current has no return conductor, its circuit is closed by electrostatic capacity, either the distributed capacity of the conductor, or capacity connected to the ends of the conductor. To produce in such a case considerable currents, either the conductor must be very long, or the frequency and E.M.F. very high.

No conductor extending parallel to the ground, as a telegraph or transmission wire can be considered as having no return, since

even if the conductor is isolated from the ground, secondary currents induced in the ground (or in the higher regions of the atmosphere) act inductively as return currents.

Hence the case of the conductor without return conductor can physically be realized only by a conductor perpendicular to the ground, as the sending and receiving antennæ of a wireless telegraph station, and even then completely only if there are no other vertical conductors, as trees, mountains, etc., which may act as inductive returns, in the space, that is on the ocean.

Since a vertical conductor is limited in length, very high frequencies are required, and so the wave is of moderate length, that is, the velocity of propagation of the magnetic (and electrostatic) field must be considered in investigating the self-induction and the mutual induction of such a conductor.

That is, the magnetic field at a distance u from the conductor, and at time t, corresponds to the current in the conductor at the time $t - t'$, where t' is the time required for the electric field to travel the distance u, that is, $t' = u/v$, where v = velocity of light. Or: the magnetic field at distance u and time t corresponds to the current in the conductor at the time $t - u/v$.

Representing the time t by angle: $\varphi = 2\pi Nt$, where N = frequency of alternating current in the conductor, and denoting

$$a = \frac{2\pi N}{v} = \frac{2\pi}{\lambda},$$

where:

λ = wave-length of electric current. (2)

The field at distance u and time angle φ corresponds to time angle $\varphi = au$, that is, lags in time behind the current in the conductor by the phase angle au.

II. Section l of Infinitely Long Straight Conductor without Return Conductor.

Let:

$$i = I \sin \varphi = \text{current, abs. units.} \tag{3}$$

The magnetic induction at distance u then is:

$$B = \frac{2I}{u} \sin (\varphi - au), \tag{4}$$

hence, the total magnetic flux surrounding the conductor, from distance r to infinity:

$$\Phi = l\int_r^\infty \frac{2I}{u}\sin(\varphi - au)du$$

$$= 2Il\left\{\sin\varphi\int_r^\infty \frac{\cos au}{u}du - \cos\varphi\int_r^\infty \frac{\sin au}{u}du\right\} \qquad (5)$$

or, substituting the symbols:

$$\left.\begin{aligned}\int_r^\infty \frac{\cos au}{u}du &= \operatorname{col} ar\\ \int_r^\infty \frac{\sin au}{u}du &= \operatorname{sil} ar\end{aligned}\right\} \qquad (6)$$

it is:

$$\Phi = 2Il\ \{\sin\varphi\ \operatorname{col} ar - \cos\varphi\ \operatorname{sil} ar\}. \qquad (7)$$

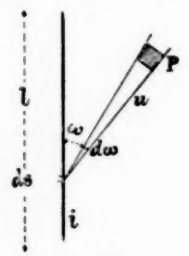

Fig. 1.

The E.M.F. consumed by this magnetic flux, or E.M.F. consumed by self-induction, then is:

$$e = \frac{d\Phi}{d\varphi},$$

hence:

$$e = 2lI\ \{\cos\varphi\ \operatorname{col} ar + \sin\varphi\ \operatorname{sil} ar\} \qquad (8)$$

and since the current is:

$$i = I\sin\varphi,$$

the E.M.F. consumed by self-induction contains an *energy component:*

$$e_1 = 2lI\ \operatorname{sil} ar\sin\varphi \qquad (9)$$

and a *wattless component:*

$$e_2 = 2l\partial\ \operatorname{col} ar\cos\varphi \qquad (10)$$

which latter leads the current by a quarter period.

The energy component e_1 gives rise to an *effective resistance:*

$$r_1 = e_1/i = 2l \text{ sil } ar. \tag{11}$$

and a *reactance*:

$$x = \left|\frac{e_2}{i}\right| = 2l \text{ col } ar. \tag{12}$$

When considering the finite velocity of propagation of the magnetic field, self-induction is thus not wattless, but contains an energy component, and can be represented by an impedance:

$$\begin{aligned} Z &= r_1 - jx, \\ &= 2l(\text{sil } ar - j \text{ col } ar), \\ &= 2l(\text{sil } ar - j \text{ col } ar)10^{-9} \text{ ohms.} \end{aligned} \tag{13}$$

The *inductance* L would then be given by:

$$\begin{aligned} L &= \frac{jZ}{2\pi N}, \\ &= \frac{l}{\pi N}(\text{col } ar + j \text{ sil } ar), \\ &= \frac{l}{\pi N}(\text{col } ar + j \text{ sil } ar)10^{-9} \text{ henrys.} \end{aligned} \tag{14}$$

The effective resistance r_1 represents the power

$$P_1 = i^2 r_1, \tag{15}$$

which is sent out by the conductor as electromagnetic radiation.

The functions:

$$\begin{aligned} \text{col } ar &= \int_r^\infty \frac{\cos au}{u}\,du, \\ \text{sil } ar &= \int_r^\infty \frac{\sin au}{u}\,du, \end{aligned}$$

or:

$$\left.\begin{aligned} \text{col } y &= \int_r^\infty \frac{\cos y}{y}\,dy, \\ \text{sil } y &= \int_r^\infty \frac{\sin y}{y}\,dy, \end{aligned}\right\} \tag{16}$$

can in general not be expressed in finite form, and so would have to be recorded in tables.

In some of the most important cases however, close approximations can be derived of these functions col y and sil y, in finite form.

III. SELF-INDUCTANCE OF CONDUCTOR.

When considering the reaction of the magnetic field on the conductor, which produces it, if:

l = length of a section of the infinitely long conductor,

r = radius of conductor,

even at extremely high frequency, r is small compared with $1/a$, hence ra a very small quantity, and then sil ar and col ar can be approximated.

The integral

$$y' = \int_0^\infty \frac{\epsilon^{-bu} \sin au}{u} du, \tag{17}$$

differentiated gives

$$\frac{dy'}{da} = \int_0^\infty \epsilon^{-bu} \cos au du. \tag{18}$$

By partial integration, it is:

$$\int \epsilon^{-bu} \cos au du = \frac{\epsilon^{-bu}(a \sin au - b \cos au)}{a^2 + b^2}.$$

Hence,

$$\frac{dy'}{da} = \frac{b}{a^2 + b^2}$$

and

$$y' = b \int \frac{da}{a^2 + b^2} + C = \tan^{-1}\left(\frac{a}{b}\right) + C. \tag{19}$$

But y' and $\tan^{-1}(a/b)$ vanish with $a = 0$ and we get $C = 0$.

The integral (17) becomes

$$y' = \int_0^\infty \frac{\epsilon^{-bu} \sin au}{u} du = \tan^{-1}\left(\frac{a}{b}\right). \tag{20}$$

In the limit case of $b = 0$, we get

$$y'' = \int_0^\infty \frac{\sin au}{u} du = \frac{\pi}{2}. \tag{21}$$

If in the integral:

$$y''' = \int_0^r \frac{\sin au}{u} du. \tag{22}$$

r is a very small quantity, expanding the sine function into a series and integrating the series, gives:

$$y''' = \int_0^r \frac{1}{u}\left(au - \frac{a^3u^3}{\underline{3}} + \frac{a^5u^5}{\underline{5}} \cdots\right) du$$

hence:

$$y''' = ar - \frac{a^3r^3}{3\,\underline{3}} + \frac{a^5r^5}{5\,\underline{5}} \cdots \tag{23}$$

Combining (21) and (23), we get

$$\int_r^\infty \frac{\sin au}{u}\, du = y'' - y''' = \frac{\pi}{2} - ar + \frac{a^3r^3}{3\,\underline{3}} - \frac{a^5r^5}{5\,\underline{5}} + \cdots \tag{24}$$

In integral calculus, we have the relation between these two definite integrals

$$\int_\delta^\gamma \frac{\cos a\theta}{\sin \frac{1}{2}\theta}\, d\theta = 2\int_{a\delta}^{a\gamma} \frac{\cos \varphi}{\varphi}\, d\varphi,$$

provided that $\gamma > \partial$, $a\gamma = \infty$ and γ and ∂ are very small.

From trigonometry, we get

$$2\{\sin\theta + \sin 2\theta + \cdots + \sin S\theta\} = \frac{2\sin \frac{1}{2}(1+S)\theta \cdot \sin \frac{1}{2}S\theta}{\sin \frac{1}{2}\theta}$$

$$= \frac{\cos \frac{1}{2}\theta - \cos (\frac{1}{2}+S)\theta}{\sin \frac{1}{2}\theta}.$$

Integrating this equation, we get

$$\int_\delta^\pi \frac{\cos (\frac{1}{2}+S)\theta}{\sin \frac{1}{2}\theta}\, d\theta = \int_\delta^\pi \cot \tfrac{1}{2}\theta\, d\theta + 2\left[\cos\theta + \tfrac{1}{2}\cos 2\theta + \cdots + \frac{1}{S}\cos S\theta\right]_\delta^\pi$$

$$= 2\, lg \frac{\sin\frac{\pi}{2}}{\sin\frac{\delta}{2}} + 2\left(-1 + \tfrac{1}{2} - \tfrac{1}{3} + \cdots + \frac{(-1)^S}{S}\right) - 2\left(\cos\partial + \tfrac{1}{2}\cos 2\partial + \cdots + \frac{1}{S}\cos S\partial\right)$$

or

$$\int_\delta^\pi \frac{\cos (\frac{1}{2}+S)\theta}{\sin \frac{1}{2}\theta}\, d\theta = -2\left\{1 - \tfrac{1}{2} + \tfrac{1}{3} - \cdots - \frac{(-1)^S}{S} - lg\, 2\right\}$$

$$-2\,lg\,\frac{2\sin\frac{\delta}{2}}{\delta}-2\,lg\,S\delta+2\left\{lg\,S-1-\tfrac{1}{2}-\tfrac{1}{3}-\cdots-\frac{1}{S}\right\}$$

$$+2\left\{1+\tfrac{1}{2}+\tfrac{1}{3}+\cdots+\frac{1}{S}-\left(\cos\delta+\tfrac{1}{2}\cos 2\delta+\cdots+\frac{1}{S}\cos S\delta\right)\right\}.$$

Now let us increase S infinitely great, and $S\delta$ remaining finite, so that δ decreases infinitely small, this combined with the preceding equation, gives

$$\operatorname*{Limit}_{S=\infty}\int_{\delta}^{\pi}\frac{\cos(\frac{1}{2}+S)\theta}{\sin\frac{1}{2}\theta}\,d\theta=2\int_{S\delta}^{S\pi}\frac{\cos\varphi}{\varphi}\,d\varphi=-2\,lg\,S\delta$$

$$+2\left\{lg\,S-1-\tfrac{1}{2}-\tfrac{1}{3}\cdots-\frac{1}{S}\right\}$$

$$+2\left\{\left[\frac{\delta^2}{\underline{2}}+\frac{(2\delta)^2}{2\,\underline{2}}+\cdots+\frac{(S\delta)^2}{S\,\underline{2}}\right]-\left[\frac{\delta^4}{\underline{4}}+\frac{(2\delta)^4}{2\,\underline{4}}+\cdots\right]+\cdots\right\},$$

or

$$\operatorname*{Limit}_{S=\infty}\int_{S\delta}^{S\pi}\frac{\cos\varphi}{\varphi}\,d\varphi=-lg\,S\delta-\left\{\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\cdots+\frac{1}{S}\right)-lg\,S\right\}$$

$$+\frac{(S\delta)^2}{2\,\underline{2}}-\frac{(S\delta)^4}{4\,\underline{4}}+\cdots$$

Let us denote the second term of the above expression by q, or

$$q=\operatorname*{Limit}_{S=\infty}\left\{\sum_{1}^{S}\frac{1}{S}-lg\,S\right\}.$$

This series has a finite value and has been computed to be

$$0.5772156\cdots.$$

Putting $S\delta=d$ and $S\pi=\infty$, we get

$$\int_{d}^{\infty}\frac{\cos\varphi}{\varphi}\,d\varphi=-q-lg\,d+\frac{d^2}{2\,\underline{2}}-\frac{d^4}{4\,\underline{4}}+\cdots.$$

Hence, we obtain

$$\int_{r}^{\infty}\frac{\cos au}{u}\,du=-q-lg\,ar+\frac{(ar)^2}{2\,\underline{2}}-\frac{(ar)^4}{4\,\underline{4}}+\cdots. \qquad (25)$$

Combining the results obtained in (24) and (25), we get

$$\Phi = 2Il\left\{\sin\varphi\cdot\left(-q-lg\,ar+\frac{(ar)^2}{2\,|\,2}-\frac{(ar)^4}{4\,|\,4}+\cdots\right)\right.$$
$$\left.-\cos\varphi\cdot\left(\frac{\pi}{2}-ar+\frac{(ar)^3}{3\,|\,3}-\frac{(ar)^5}{5\,|\,5}+\cdots\right)\right\}. \quad (26)$$

With the dimension of common wire, we can neglect all terms containing ar except $lg\,ar$, and we have

$$\Phi' = -2Il\left\{(q+lg\,ar)\cdot\sin\varphi+\frac{\pi}{2}\cos\varphi\right\}, \quad (27)$$

or

$$\Phi' = -2Il\sqrt{(q+lg\,ar)^2+\frac{\pi^2}{4}}\sin(\varphi-\omega), \quad (28)$$

where

$$\tan\omega = \frac{\pi}{2(q+lg\,ar)} \quad (29)$$

and

$$q = 0.5772156\cdots. \quad (30)$$

Therefrom follows: (13)

$$Z = 2l\left\{\frac{\pi}{2}+j(q+lg\,ar)\right\}10^{-9}$$
$$= 2l\left\{\frac{\pi}{2}-j\left(lg\frac{1}{ar}-q\right)\right\}10^{-9}\text{ ohms} \quad (31)$$

and, (14)

$$L = \frac{l}{\pi N}\left\{-(q+lg\,ar)+j\frac{\pi}{2}\right\}10^{-9}$$
$$= \frac{l}{\pi N}\left\{\left(lg\frac{1}{ar}-q\right)+j\frac{\pi}{2}\right\}10^{-9}\text{ henrys} \quad (32)$$

as the effective impedance, and inductance, respectively, of a section l of an infinitely long conductor at frequency N, where:

$$a = \frac{2\pi N}{v}$$

and

v = velocity of light.
$q = .5772156\cdots$

It follows herefrom:

The power radiated by the conductor, section l, as electromagnetic wave:

$$P = i^2 r_1$$

$$= 2l\frac{\pi}{2} i^2$$

$$= \pi l i^2 \qquad (33)$$

is independent of the frequency N, and the conductor dimensions, as the radius r, as long as ar is negligable, that is, $2\pi Nr$ small compared with the velocity of light v, or $2\pi r$ small compared with the wave-length λ.

Instance:

$$l = 200 \text{ ft} = 6{,}000 \text{ cm.}$$

$$r = .4'' = 1 \text{ cm.}$$

$$N = 400{,}000 \text{ cycles,}$$

hence

$$\lambda = 2500 \text{ ft.}$$

$$ar = 85 \times 10^{-6}$$

and

$$Z = (18.9 - 105.5j)10^{-6} \text{ohms.}$$

At

$$i = 1000 \text{ amps.:}$$

$$P = 18.9 \text{ watts.}$$

IV. Self-Inductance of Straight Conductor of Finite Length l.

Let

$i = I \sin \varphi$ = current.

l = length of the wire.

$a/2\pi N$ = reciprocal of the velocity of the propagation of the electromagnetic wave.

ds = equivalent current element on the axis of the wire.

u = vector drawn from ds to the point P outside the wire.

ω = the angle between the vector u and the current i.

The magnetic induction due to the current element at any point in free space is represented by

$$B = \frac{I \sin(\varphi - au) \cdot \sin\omega \cdot ds}{u^2}.$$

The total flux of magnetic induction around the wire is given by the following integral

$$\Phi = \int_0^l \int_0^\pi \int_{r/\sin\omega}^\infty \frac{I \sin(\varphi - au) \cdot u \cdot \sin\omega}{u^2} du\, d\omega\, ds \qquad (34)$$

or

$$\Phi = I \int_0^l \int_0^\pi \int_{r/\sin\omega}^\infty \frac{\sin(\varphi - au) \cdot \sin\omega}{u} du\, d\omega\, ds. \qquad (35)$$

The first integral of the above equation is just the same form as treated in the preceding case, and we have

$$\Phi = I \int_0^l \int_0^\pi \left\{ \sin\varphi \cdot [-q - lg\, ar + lg(\sin\omega)] - \cos\varphi \cdot \frac{\pi}{2} \right\} \sin\omega\, d\omega\, ds. \qquad (36)$$

Consider the integral

$$\int_0^\pi \sin\omega \cdot lg(\sin\omega) d\omega,$$

we have

$$X \equiv \int_0^\pi \sin\omega \cdot lg(\sin\omega) d\omega = 2 \int_0^{\pi/2} \sin\omega \cdot lg(\sin\omega) d\omega$$

and

$$Y \equiv \int \sin\omega \cdot lg(\sin\omega) d\omega = \cos\omega - \cos\omega \cdot lg(\sin\omega) + \tfrac{1}{2} lg \frac{1 - \cos\omega}{1 + \cos\omega}.$$

Hence, we obtain

$$X = 2L \lim_{\epsilon=0} [Y]_{\epsilon/2}^{\pi/2} = 2L \lim_{\epsilon=0} \left\{ lg\, 2 - 1 + \left(lg \frac{\epsilon}{2} - lg \frac{\epsilon}{2} \right) \right\}$$

$$= 2(lg\, 2 - 1),$$

or

$$\int_0^\pi \sin\omega \cdot lg(\sin\omega) d\omega = 2(lg\, 2 - 1).$$

Consequently, we get

$$\Phi = 2I \int_0 \left\{ -(q + lg\, ar + 1 - lg\, 2) \sin\varphi - \frac{\pi}{2} \cos\varphi \right\} ds,$$

or

$$\Phi = 2Il\left\{\left(lg\frac{1}{ar} - q - 1 + lg2\right)\sin\varphi - \frac{\pi}{2}\cos\varphi\right\}$$

$$= 2Il\left\{\left(lg\frac{1}{ar} - s\right)\sin\varphi - \frac{\pi}{2}\cos\varphi\right\} \qquad (37)$$

where

$$s = q + 1 - lg2 = .8841\cdots$$

or

$$\Phi = 2Il\sqrt{\left(lg\frac{1}{ar} - s\right)^2 + \left(\frac{\pi}{2}\right)^2}\sin(\varphi - \omega) \qquad (38)$$

where

$$\tan\omega = \frac{\pi}{2\left(lg\frac{1}{ar} - s\right)} \qquad (39)$$

hence

$$Z = 2l\left\{\frac{\pi}{2} - j\left(lg\frac{1}{ar} - s\right)\right\}10^{-9} \text{ ohms}, \qquad (40)$$

and

$$L = \frac{l}{\pi N}\left\{\left(lg\frac{1}{ar} - s\right) + j\frac{\pi}{2}\right\}10^{-9} \text{ henrys}. \qquad (41)$$

Instance:

Length of conductor, $l = 200$ ft. $= 6000$ cm.

Diameter of conductor, $2r = .8'' = 2$ cm.

Conductor grounded at bottom, connected at top to capacity about three times that of the conductor, so as to reduce its frequency of oscillation to $N = 400{,}000$ cycles. This gives $Z = (18.9 - 102j)10^{-6}$ ohms. At $i = 1{,}000$ amps., $P = i^2r_1 = 18.9$ watts.

V. Mutual Inductance between two Finite Straight Conductors at Considerable Distance from Each Other.

This problem is of interest in relation to wireless telegraphy, in that it investigates one component, the electromagnetic, of the electric field which transmits the message between sending and receiving station.

The distance between the two conductors is supposed to be very great compared with the lengths of the wires, and the angles subtended by one of the wires to any point in another wire may be considered equal to each other.

Let :

l and l_1 = lengths of the straight conductors in space.

D = length of any vector connecting the wires.

α and α_1 = angles between D and the wires.

As we have assumed D to be very great compared with l and l_1, the angles α and α_1 may be considered remain constant independent of D.

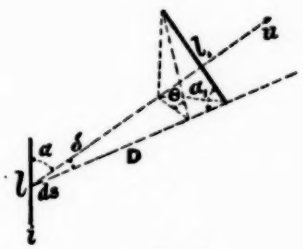

Fig. 2.

Also, let us denote by δ the angle subtended by any point in l carrying electric current i to the projection of l_1 upon the plane determined by l and D; and θ the angle between the planes determined by the vector D and each of the wires l and l_1.

The projection of l_1 upon the plane of l and D can be decomposed into two components, one parallel to and another perpendicular to the vector D; thus, we get

$$l_1 \cos \alpha_1 = \text{parallel component.}$$
$$l_1 \sin \alpha_1 \cos \theta = \text{normal component.}$$
$$\delta = \frac{l_1 \sin \alpha_1 \cos \theta}{D} .$$

Hence, we get the following integral for the expression of the total flux of the magnetic induction which has just passed over the conductor l_1 at the instant t.

$$\Phi = \int_0^l \int_D^\infty \frac{I \sin (2\pi N t - au) \cdot \sin \alpha \cdot u \cdot \delta}{u^2} \, du \, ds$$

or

$$\Phi = \frac{Il_1 \sin \alpha_1 \sin \alpha \cos \theta}{D} \int_0^l \int_D^\infty \frac{\sin (\varphi - au)}{u} \, du \, ds.$$

Let us suppose that the distance D is very great compared with the lengths of conductors l and l_1 and the wave-length λ.

Putting $D = \lambda(m + s)/4$, where m is a large positive integer and s the positive fractional part less than one, we have

$$X \equiv \int_D^\infty \frac{\sin(\varphi - au)}{u} du$$

$$= \int_D^\infty \frac{\sin\varphi\cos au}{u} du - \int_D^\infty \frac{\cos\varphi\sin au}{u} du$$

$$= \sin\varphi \left[\int_D^{\lambda(m+1)/4} + \int_{\lambda(m+1)/4}^{\lambda(m+2)/4} + \cdots \right] \frac{\cos au}{u} du$$

$$- \cos\varphi \left[\int_D^{\lambda(m+1)/4} + \int_{\lambda(m+1)/4}^{\lambda(m+2)/4} + \cdots \right] \frac{\sin au}{u} du$$

(i) $m = 2n$

$$X_1 \doteqdot (-1)^n \left(1 - \sin\frac{s\pi}{2}\right) \frac{\sin\varphi}{\frac{\lambda}{2}na} + (-1)^n \left(1 - \cos\frac{s\pi}{2}\right) \frac{\cos\varphi}{\frac{\lambda}{4}(2n+1)a}$$

$$- (-1)^n \frac{\sin\varphi}{\frac{\lambda}{4}a} \left\{ \frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} \cdots \right\}$$

$$- (-1)^n \frac{\cos\varphi}{\frac{\lambda}{4}a} \left\{ \frac{1}{n+\frac{1}{2}} - \frac{1}{n+1+\frac{1}{2}} + \frac{1}{n+2+\frac{1}{2}} \cdots \right\}.$$

(ii) $m = 2p + 1$

$$X_{11} \doteqdot (-1)^p \frac{\sin\varphi}{\frac{\lambda}{4}(2p+2)a} \left(1 - \sin\frac{s\pi}{2}\right)$$

$$+ (-1)^p \frac{\cos\varphi}{\frac{\lambda}{4}(2p+1)a} \left(1 - \cos\frac{s\pi}{2}\right)$$

$$- (-1)^p \frac{\sin\varphi}{\frac{\lambda}{4}a} \left\{ \frac{1}{p+1} - \frac{1}{p+2} + \cdots \right\}$$

$$+ (-1)^p \frac{\cos\varphi}{\frac{\lambda}{4}a} \left\{ \frac{1}{p+1+\frac{1}{2}} - \frac{1}{p+2+\frac{1}{2}} + \cdots \right\}.$$

These results have already a relative error of $(\lambda/4D)^2$ order and the summation of the fractional series can be obtained only in approximate ways.

It can be shown that the summation of the series,

$$S \equiv \frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2} - \frac{1}{n+3} + \cdots \equiv \sum_{p=0}^{\infty} \frac{(-1)^p}{n+p},$$

can be represented by $\frac{1}{4}\log(n^2-1)/n(n-2)$ with a relative error of $(1/n)^3$ order.

For the sake of simplicity, we shall follow another method of the approximate calculation.

Let us consider the following series

$$S_0 = \frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2} \cdots$$

$$S' = \frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} \cdots,$$

then we have the following relations

$$S > S'$$

$$S > \frac{1}{(n+1)^2} + \frac{1}{(n+3)^2} + \frac{1}{(n+5)^2} + \cdots$$

$$S' < \frac{1}{(n+1)^2} + \frac{1}{(n+3)^2} + \frac{1}{(n+5)^2} + \cdots$$

$$S < \frac{1}{n^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+4)^2} + \cdots$$

$$\therefore \quad S - S' < \frac{1}{n^2},$$

moreover we have

$$S + S' = \frac{1}{n}.$$

Therefore, neglecting the small quantity $(1/n)^2$, we get

$$S \doteqdot \frac{1}{2n}.$$

Substituting this approximate result in the preceding equations, we get the generalized formulæ

$$\int_{D=\lambda(m+S)/4}^{\infty} \frac{\sin(\varphi - au)}{u}\, du \doteqdot -(-1)^m \frac{\cos(\varphi - aD)}{aD + \pi/2}.$$

Hence, we get

$$\Phi \doteqdot (-1)^{m+1} \frac{Ill_1 \sin a \sin a_1 \cos \theta}{D(aD + \pi/2)} \cos (\varphi - aD)$$

where

$$D = \frac{\lambda}{4}(m + s) \text{ and } 0 < s < 1$$

and the formula shall give the result with a relative error of $(1/m)^2$ order.

Taking the differential coefficient of Φ with respect to t, we obtain the expression for the induced E.M.F.

$$e = -\frac{d\Phi}{dt} \doteqdot (-1)^{m+1} \frac{2\pi N Ill_1 \sin a \sin \alpha_1 \cos \theta}{D(Da + \pi/2)} \sin (\varphi - aD),$$

or

$$e \doteqdot (-1)^{m+1} \frac{vIll_1 \sin a \sin a_1 \cos \theta}{D\left(D + \frac{\lambda}{4}\right)} \sin \frac{2\pi}{\lambda}(vt - D),$$

where v is the velocity of the propagation of the electromagnetic wave in the air and λ the wave-length.

Writing the equation in another form, we get

$$e \doteqdot (-1)^{m+1} \left(1 - \frac{\lambda}{4D}\right) \frac{vIll_1 \sin a \sin a_1 \cos \theta}{D^2} \sin \frac{2\pi}{\lambda}(vt - D).$$

Instances:

$l = l_1 = 200$ ft. $\doteqdot 6100$ cm.

$r = .4$ inch $\doteqdot 1$ cm.

$N = 4 \times 10^5$.

$I/\sqrt{2} = 1000$ amp.

$D_1 = 10$ miles $= 1.61 \times 10^6$ cm.

$D_2 = 200$ miles $= 3.22 \times 10^7$ cm.

$D_3 =$ Quadrant of the earth $\doteqdot 6214$ miles $\doteqdot 10^9$ cm.

The induced E.M.F. become

(i) $$\frac{E_1}{\sqrt{2}} = 0.427 \text{ volt.}$$

(ii) $$\frac{E_2}{\sqrt{2}} = 1.08 \text{ millivolts.}$$

(iii) $$\frac{E_3}{\sqrt{2}} = 1.12 \text{ microvolts.}$$

The preceding investigation applies only to the electromagnetic component of the electric field of a straight conductor. Similar equations, however, would be derived in an analogous manner for the electrostatic component of the electric field, that is, for the capacity.

From the relation between the inductance L and the capacity C of a finite length l of an infinitely long conductor.

$$LC = \frac{l^2}{v^2}$$

where v = velocity of light; it follows, that when neglecting the velocity of propagation of the electric field in space, the capacity of a finite length of an infinitely long conductor without return conductor equals zero:

$$C = 0$$

since:

$$L = \infty .$$

Considering the finite velocity of propagation of the electric field in space, L becomes finite, but a function of the frequency of the alternating current, and so the capacity C also becomes finite, and a function of the frequency, which may be evaluated by a similar investigation.

A STUDY OF THE PROPAGATION AND INTERCEPTION OF ENERGY IN WIRELESS TELEGRAPHY. PART I.

BY CHARLES A. CULVER.

PURPOSE.

FROM time to time our own and various foreign governments have issued numerous patents covering various types of antennæ or aërial systems, which have been designed for the purpose of radiating or receiving electromagnetic waves. While there has been some quantitative work done on the efficiency of these various types, the investigations have been conducted largely along commercial lines. The writer is not aware that any extended quantitative research in this direction has been carried out under conditions permitting of delicate and highly accurate measurement. Aside from the practical bearing which the results of such a research might have, it was believed that a study of the action of these various types might possibly yield data which would throw some light upon one or more of the undetermined theoretical problems at present existing in this field. With this end in view it was *the purpose of this investigation to make a study of the relative efficiency of several different types of receiving systems when used under various conditions.* In so far as this paper treats of aërials or antennæ it has to do with conditions at the receiving station only. In a future paper we hope to investigate several radiating systems in a similar manner.

APPARATUS EMPLOYED.

Sending Station. — The equipment at the power station consisted of a transformer, T, Fig. 1, a, two inductance coils, L_1 and L_2, a spark gap, G_1 and a battery of six Leyden jars, C_1. A single insulated wire [1] ten meters in length served as a radiating system.

[1] Whenever wire is referred to hereafter, unless otherwise specified, insulated copper wire No. 10 bearing the trade name "Fire and Water Proof" is meant.

Seven and eight tenths meters of this wire were supported in vertical position by means of a specially constructed high tension insulator. The transformer employed was rated to deliver 25,000 volts when operating on a 104 volt 60 cycle circuit. The voltage, however, was cut down to about 5,000 by means of an adjustable rheostat in

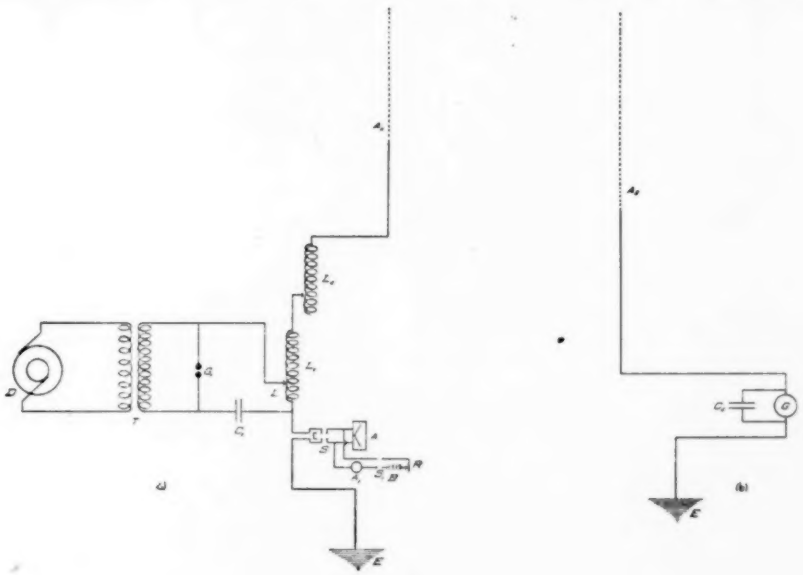

Fig. 1.

series with the primary. Power was secured from commercial mains. The Leyden jars composing the capacity, C_1, were specially constructed for high tension work and were worked in parallel, their combined capacity being 0.00837 M-F. The two inductance coils, L_1 and L_2, consisted of 23 turns of No. 9 bare copper wire (B.S.G.) wound on rectangular wooden frames 30.5 cm. square, the pitch of the winding, being 1.3 cm. One of these coils, L_1, served as on auto-transformer and the other, L_2, as an auxiliary tuning coil. The spark gap, G_1, was located on the top of the coil, L_1, and consisted of two extremely hard and highly polished steel balls 2.54 cm. in diameter, resting on the top of two short upright pieces of brass tubing 1.3 cm. internal diameter. One of these brass supports was adjustable, thus permitting the length of the spark gap to be varied. A spark length of 1 or 2 mm. was employed. Such an

arrangement is probably as satisfactory as any form of gap for the reason that such steel balls are very refractory and may be readily changed or polished. Signals were made by opening and closing a switch in the primary circuit of the transformer, T. The power in the commercial mains from which our energy supply was taken varied between 3 and 4 per cent. in voltage. Various forms of earth connections were tried and the one ultimately employed was not used because it permitted the greatest amount of energy to be radiated from the sending station, for it did not, but for other reasons which will become obvious as we proceed. A hole $2 \times 4 \times 6$ feet was excavated in the earth. In the bottom of this hole a piece of gas pipe, 6 feet long, was driven until but 2 or 3 inches remained above the bottom of the hole. A sheet of galvanized iron 2×8 feet, having a hole in the middle large enough to admit the gas pipe, was placed in the excavation. This covered the bottom of the hole and extended upward on either of two sides for some 2 or 3 feet. The gas pipe passed through the hole in the middle of the metal sheet, extending above the same for 2 or 3 inches. Bare copper wire No. 9 (B.S.G.) led from the auto-transformer to earth, being soldered to the plate and also to the pipe. Such an arrangement was found to work reasonably well, though not so satisfactorily as a "ground" made by soldering the earth wire to gas or water pipes in the building. Both the ground and antenna wires were led out of the building through heavy porcelain insulators.

In order to determine when the two oscillating systems at the sending station were in resonance with each other, a specially constructed hot-wire ammeter was inserted between the inductance, L_1, and the earth. Fleming and others have pointed out that the ordinary commercial hot-wire ammeter will not give correct readings when operating on a high frequency circuit. This is due of course to the fact that with any instrument operating upon the shunt principle, the amount of current which will pass through the instrument proper is a function of the frequency. Since in the process of bringing two parts of any high frequency alternating circuit into resonance the frequency of one or both of the related circuits changes, therefore an instrument to give correct readings, even though only maximum current is desired, must be independent of the frequency

and wave form. In this investigation an instrument was constructed along the lines suggested by Professor Fleming.[1]

Referring to Fig. 2, WW_1 is a No. 34 copper wire 32 cm. long, stretched between two binding posts, BB_1. This wire is drawn taut by a silk thread, TT_1, 16 cm. in length, fastened to its middle part

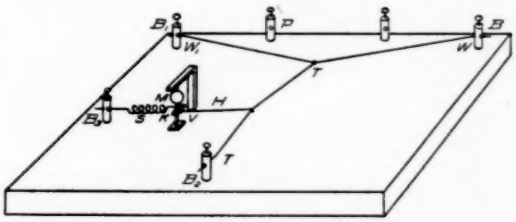

Fig. 2.

by means of a small double hook made of glass, the copper wire slipping freely through the glass hook. The end, T_1, of the above thread is fastened to a short piece of heavy brass wire which is held in place by a binding post, B_2. V is a short steel shaft having conical points at either end and supported by suitable brass bearings fastened to the base of the instrument, which is of wood. This shaft carries a mirror, M, fastened to it by any convenient means. The shaft carries a short brass projection, K, having two small holes near its free extremity. A piece of fine piano wire, H, 3 cm. long having a loop at either end connects this brass lever to the middle of the thread, TT_1. The system thus formed is kept taut by means of spring, S, attached to the brass lever, K, and to a short piece of heavy brass wire which is held in place by means of a binding post, B_3. Any expansion of the wire, WW_1, due to current passing through the same will result in a movement of the mirror, M, which may be read by means of telescope and scale in the usual manner. The tension on the wire and the normal position of the mirror is modified by means of the adjustments possible at B_2 and B_3. A light wooden box enclosed the working parts of the instrument. By suitable choice of the wire, WW, various ranges and degrees of sensibility can be secured. Such an instrument gives the root-mean-square value of the current passing through it, and is independent

[1] The Principles of Electric Wave Telegraphy, p. 142.

of the frequency and wave form. Fig. 3 shows a typical calibration curve of the instrument, made by means of a standard Weston instrument and direct current. A series of tests extending over several days, under varying conditions of temperature and humidity, showed that the instrument held its calibration to a fair degree of accuracy. However, to avoid any possible changes due to external conditions we did not depend upon the calibration curve but used the instrument as a comparator. A reference to Fig. 2 will indicate how this was accomplished. The hot-wire ammeter was so arranged that it

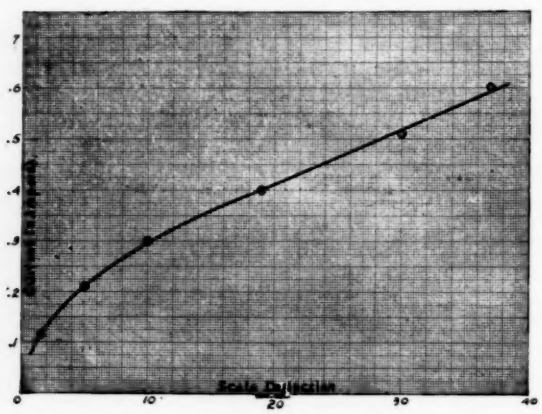

Fig. 3.

could be switched into the oscillating circuit at the same point at all times. After the mean-square value of the oscillating current had been read the ammeter was switched out of the oscillating circuit and into a direct current circuit in series with a finely adjustable rheostat and a standard Weston ammeter. The direct current was then adjusted until a deflection of the hot-wire instrument was secured equal to that caused by the oscillating current. The reading of the Weston instrument then gave the root-mean-square value of the high frequency current. The instrument was utilized not only to determine the conditions of maximum current in the inductance circuit but also to serve as a check on the constancy of the energy output at the sending station whenever it was impracticable to repeat any given set of measurements. After the two circuits at the sending station were brought into resonance the relation of the parts was not changed during the remainder of the experiments.

Receiving Station.

The receiving station was equipped with various forms of antennæ; two adjustable oil condensers, C_2, arranged in parallel and shunted about the energy-measuring instrument; an oscillating current galvanometer, G, which served to measure the energy intercepted by the various receiving systems; an earth connection, E, similar to that described for the sending station. Fig. 1, *b*, illustrates the connections at the receiving station. The oil condensers consisted of two sets of sectorial-shaped brass plates, one set of which revolved about an axis set near one edge of the movable plates and close to the fixed sectors. A pointer attached to the axis moved over a scale graduated in circular degrees. Several such condensers were at our disposal, having been constructed from plans and specifications given by Clyde C. Swayne in an article which appeared in the American Electrician for September, 1905. Each condenser had a maximum capacity of .00211 M-F., and a minimum of .000771 M-F. Such condensers were found to serve admirably for delicate tuning in wireless circuits.

The energy-measuring instrument was an oscillating current galvanometer modelled along the lines suggested by Fleming and developed by Northrup[1] and Pierce.[2] In general the mechanical construction did not differ materially from the instrument now sold by the Leeds & Northrup Co., except that only one coil was used and the damping magnet was omitted. In detail the instrument as used in this investigation consisted of a small coil composed of 45 turns of No. 36 D.S.C. copper wire wound on a hollow hard rubber bobbin 7 mm. internal diameter. Directly in front of this coil was suspended by a very fine quartz fiber a small silver disc 6 mm. in diameter and .2 mm. thick. The suspension fiber was fastened to the metal disc by shellacing both to a delicate glass stem. The glass stem also carried a tiny mirror. The system thus composed was enclosed in a square hard rubber case having glass windows front and back. The suspension tube was of brass. Brass levelling screws completed the instrument. A non-inductive shunt might have been employed to vary the sensibility of the galvanometer;

[1] Electrical World, December 18, 1898.
[2] Physical Review, Vol. 19, p. 202.

however, only one degree of sensibility was used throughout the experiments. Northrup and Pierce have shown that for small angles the deflection of such an instrument is proportional to the square of the current. The deflections were read by means of telescope and scale placed at a distance of 83 cm. from the galvanometer.

The two stations were 47 meters apart, the sending antenna being located on the north side of one building and the various receiving systems on the south side of an adjacent building, the two buildings constituting the Randall Morgan Laboratory thus being located between the two stations.

WAVE-LENGTH MEASUREMENTS.

In order to measure the two wave-lengths emitted by the transmitting station, a wave-measuring apparatus was set up consisting of one of the adjustable oil condensers before referred to; a small inductance coil 21 cm. in diameter, carrying 9 turns of low resistance wire; a Duddell thermo-galvanometer in series with the condenser and inductance.[1] The inductance of the small helix and leads to the same measured by the Fleming-Anderson method was .0642 milli-henrys. This wave-measuring system was set up in the same room with the sending apparatus about 2 m. distant from the same. By means of such an arrangement the wave-length could be measured without the wave-meter circuit exerting any influence upon the system whose wave was being tested. The action of the thermo-galvanometer indicated that it would have been possible to perform the experiment with the wave-meter at even a much greater distance from the transmitting system. The wave-meter circuit was brought into resonance with the sending circuit by adjusting the oil condenser. It was found possible to secure very sharp resonance, a change of .5 of 1 per cent. in the capacity resulting in a falling off of at least 10 per cent. in the current as indicated by the galvanometer. This meant that a change in capacity of nine millionths of

[1] As this investigation is in progress we note the appearance of a paper by Professor Pierce (PHYSICAL REVIEW, Vol. 24, p. 152) in which the author describes the use of a similar device for measuring frequencies. Professor Pierce employs an oscillation galvanometer where we use the Duddell instrument. We believe the thermo-galvanometer to be somewhat superior for this purpose in that it is more dead beat and at the same time is amply sensitive for the purpose.

a microfarad was readily detectable. Advantage was taken of this fact later in measuring certain small values of capacity by means of an obvious substitution method. As was to be expeeted, the galvanometer indicated two current maxima, one with .001464 M-F., another with .001079 M-F.

Fleming has developed the working formula

$$n = \frac{5 \times 10^6}{\sqrt{LC}},$$

where n represents the frequency, L the inductance in centimeters and C the capacity in microfarads. If we substitute in this equation the well-known relation

$$n = \frac{V}{\lambda},$$

where V is the velocity of propagation and λ the wave-length, we obtain the convenient expression

$$\lambda = 6 \times 10^3 \sqrt{LC}.$$

Substituting in this equation the values given above for L and C we get

$$\lambda_1 = 6 \times 10^3 \sqrt{64{,}200 \times .001464} = 581.7 \text{ meters},$$

$$\lambda_2 = 6 \times 10^3 \sqrt{64{,}200 \times .001079} = 499.2 \text{ meters}.$$

Thus it will be seen that the wave-lengths employed were comparable with those used for commercial purposes, the wave-length of the navy being 425 m.

Effects of Earth Resistance.

Preliminary Experiments. — During the course of some preliminary experiments conducted at the beginning of this investigation, it was observed that with the receiving apparatus grounded by connecting to a piece of buried wire netting and the sending apparatus grounded to gas or steam pipes, the deflection of the galvanometewas four times what it was when the sending station was grounded to similar pieces of netting. When the receiving station was also grounded to pipes, the transmitting station being earthed through piping, the deflection at the receiving station was ten times what it

was when both stations were earthed to buried netting. If the transmitting station were grounded to the plate and the receiving apparatus grounded first to the plate and then to the pipes in turn, little, if any, difference in the amount of energy received resulted. From these experiments it is apparant that the earth connections play a highly important part in the transmission of energy in wireless telegraph operations, particularly the "ground" at the transmitting station. While this fact is not new such marked differences as those observed when shown by careful measurements led the writer to try some experiments having to do with the effect earth resistance *between* stations might possibly have upon the quantity of energy transmitted. Accordingly the two stations were grounded to the two large galvanized iron earth plates and a wire run between the two stations, connecting to the earth leads where they enter the ground. Between the two stations this wire was insulated from the earth. In series with this wire was an adjustable non-inductive resistance. Thus there was arranged a variable resistance in parallel with the earth. With a constant supply of energy being radiated from the sending station, readings were made at the the receiving station with the parallel-earth-wire *disconnected* at both ends and then again with the variable resistance *connected* in parallel with the earth. While the energy received as shown by the relative deflections of the galvanometer was greater as the earth resistance was decreased, the results were not wholly concordant.

The experiment was next tried of placing several wires between the two stations so arranged that various combinations might be introduced in parallel with the earth. These wires were insulated from the earth except at the points where they connected to the earth leads where these leads entered the ground. To reduce the amount of energy received in order to bring the same within the range of the galvanometer without employing a shunt, a vertical receiving antenna 1.3 m. long was employed. The system thus constituted was brought into resonance with the sending station by adjusting the oil condensers shunted about the galvanometer. (This method of tuning was employed in all the experiments of this research.) By means of a Wheatstone bridge, a buzzer and a telephone the resistance of the earth between the two earth plates

was measured. Similarly the resistance between these two points was measured when 1, 2, 3, . . . 6 wires were in parallel with the earth. Readings were also taken of the relative amounts of energy received under these various conditions. The amount of energy received increased materially as the earth resistance between the two stations was decreased. Table I. embodies the results of this experiment. In Fig. 4 the earth resistance between the two stations is plotted against galvanometer deflection (current square).

TABLE I.

No. of Wires in Parallel with Earth.	Resistance Between Earth Plates, Ohms.	Gal. Deflection (Current Square).	No. of Wires in Parallel with Earth.	Resistance Between Earth Plates, Ohms.	Gal. Deflection (Current Square).
0	90.0	4.0	4	17.0	117.0
1	74.7	38.5	5	14.0	128.0
2	34.7	78.0	6	11.7	140.0
3	22.9	102.0			

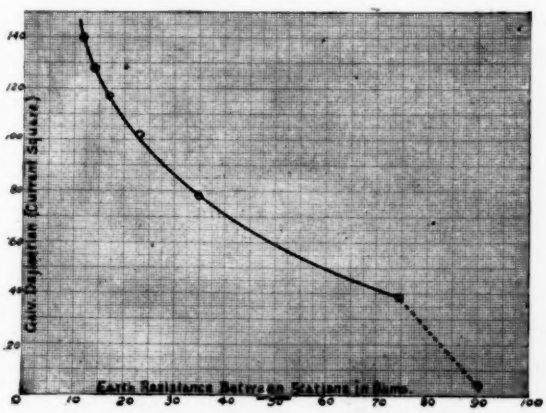

Fig. 4.

To further test the matter of earth resistance the wires above referred to were removed and a bare copper wire, No. 32, was strung out loosely on the damp ground and connected to the earth leads as in the above case. Immediately before this latter wire was put in place a mean reading was secured of the energy received, and as it was not practicable to repeat the test, a note was made of the reading of the antenna current at the sending station. When the

wire was placed upon the ground as indicated above a mean reading of the received energy was made, and the galvanometer deflection proved to be 4.7 times the value when no wire connected the stations. The ratio between the current received in each case was nearly 2.2. The hot-wire ammeter at the sending station read slightly lower at the close of the experiment than at the beginning. Reference to the above facts will be made later in the paper.

HORIZONTAL ANTENNÆ.

An antenna was next constructed by supporting a wire 14.65 m. long in a horizontal position by means of porcelain insulators fastened to stakes driven in the ground. The mean height of this

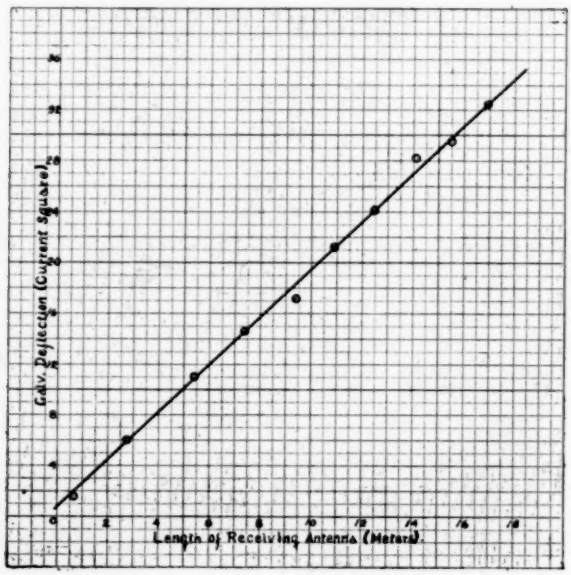

Fig. 5.

wire above the earth was 7 m. Since the receiving apparatus was located in a basement room, the permanent horizontal portion of antenna (*H*, Fig. 6) which led from the table containing the receiving apparatus to the exterior of the building was on a line with the horizontal antenna above referred to, thus making a total length of 16.8 m. Since it has been shown by Marconi, Fleming and others that such a horizontal antenna is most efficient when its free

end is pointing directly away from the sending station, the above wire was placed in this position, the direction being determined first approximately by experiment and afterwards by means of a theodolite.

After the system thus established was brought into resonance with the sending station, which in this case was grounded to gas piping, the receiving station being connected to the earth plate, readings of the galvanometer deflections were taken as various portions of the horizontal receiving antenna were cut off. After any portion of the antenna was removed the system was again tuned to resonance. Table II. gives the result of this experiment. In Fig. 5 galvanometer deflections are plotted against length of receiving antenna. Though some of these points do not lie exactly on the curve, yet other data taken confirms the fact that *in the case of a horizontal antenna under the conditions with which we worked the square of the current received is proportional to the length of the antenna.*

TABLE II.

Shunted Capacity Necessary to Effect Resonance.	Length of Horizontal Receiving Antenna.	Gal. Deflection.	Shunted Capacity Necessary to Effect Resonance.	Length of Horizontal Receiving Antenna.	Gal. Deflection.
0.001559 m-f.	16.80 meters.	32.5	0.001600 m-f.	9.40 meters.	17.2
0.001571 "	15.50 "	29.5	0.001609 "	7.40 "	14.7
0.001581 "	14.20 "	28.2	0.001613 "	5.40 "	11.0
0.001591 "	12.50 "	24.2	0.001615 "	2.75 "	6.0
0.001593 "	10.90 "	21.2	0.001617 "	0.65 "	1.6

EFFECT OF ORIENTATION ON VARIOUS FORMS OF ANTENNÆ.

Capacity-areas. — To test the effects of orientation on different forms of antenna several types were constructed and tested. The first form experimented with consisted of a capacity-area 1 m. square made by tacking a piece of sheet tin to a wooden frame and supporting this by means of a metal rod passing through porcelain insulators attached to the supporting frame-work. The rod serving as an axis was supported by a wooden tripod. 1.05 m. of wire, nearly horizontal, connected this capacity-area to the permanent antenna lead (*H*, Fig. 6), the wire being soldered to the middle of the bottom edge of the tin. This capacity-area was supported at a

height of 70 cm. above the ground at the point where the tripod stood and at a height of 16 cm. above the mean level of the ground between the two stations. By thus placing the area close to the ground it was possible to eliminate any effect due to any vertical wire leading to the same. Fig. 6 shows the general arrangement of parts in this and several succeeding experiments. The arrange-

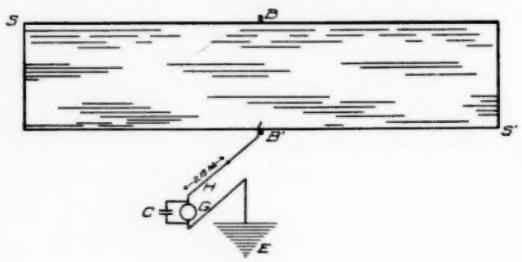

Fig. 6.

ment indicated permitted of rotating the capacity-area about a vertical axis. After bringing the system into resonance with the sending station, it was found that *the amount of energy received was the same for all planes of orientation about a vertical axis.*

The same area was next arranged in such a manner that orientation about a horizontal axis was possible. Other conditions were the same as in the previous case except that the area was supported 25 cm. above the mean level of the ground, and a vertical wire 75 cm. in length connected the middle of the tin sheet to the short horizontal lead *H* before mentioned.

Readings of the energy received were made for various angles as the capacity-area was rotated about its horizontal axis. The mean of a number of readings showed that the area when in a vertical position produced a galvanometer deflection 5.6 per cent. (current 2.7 per cent.) greater than when horizontal. The difference between the maximum and minimum values is so small that it was not deemed of value to insert readings for intermediate angles.

Since the above results obtained on rotating the capacity-area about a vertical axis do not agree with the results obtained by Dr. De Forest,[1] it was decided to test the matter further. Accordingly

[1] "With a mechanism of this character having a collecting-screen 6 × 15 feet in size, I have been able to locate with certainty a transmitting-station seven miles distant within

a "collecting-screen" 4.17 × .71 m. was made of tin plates soldered together and fastened to suitable frame-work. The general arrangement of support was the same as for the smaller area (see Fig. 6), the center of the screen being 90 cm. above the earth at the point of support. A wire nearly horizontal, 2.67 m. in length, connected the middle of the lower edge of the metal surface to the lead, *H*. The screen was free to rotate about a vertical axis at its middle point.

After resonance was effected readings of energy received were made for various angles of rotation. *In this case a difference of 4.6 per cent. in the galvanometer deflection (current 2.4 per cent.) was found to exist, the greater amount of energy being received when the collecting-screen was broadside on to the waves.* Intermediate values are not recorded here. The above difference is by no means as great as that recorded by the investigator above referred to. It is to be remembered however that Dr. De Forest used the relative loudness of sounds in a telephone receiver as judged by the human ear as a means of determining the amount of energy received for various positions of his collecting-screens. It can scarcely be said that such equipment permitted of accurate quantitative measurements. Nevertheless it is possible that the same law does not obtain for both short and long distances.

Helix.

To test the effects of orientation upon an open circuited coil of large diameter when utilized as a receiving system a helix was constructed by winding two complete turns of wire about a wooden frame 1.5 m. square. This helix had an inductance of .040982 milli-henrys (Fleming-Anderson method). The helix thus made was supported and insulated as in the case of the small capacity-area, the free ends of the wire being at the middle of the lower edge of the coil. 1.7 m. of wire connected one end of the helix to the lead, *H*. The helix was so arranged as to permit of rotation about a central vertical axis. When resonance was secured in the usual manner energy readings were noted for various planes of orientation.

10 degrees, the transmitting-station being one designed for signal-transmitting purposes." U. S. Patent No. 771,818, October 11, 1904, line 94.

It was found that such an open circuited helix collected an equal amount of energy in all positions.

The experiment was next tried of breaking the ground connection at the receiving station and in its place making a connection to the remaining free end of the helix above described, thus making a closed oscillating system without earth connections. In the case last outlined the galvanometer showed a deflection of 87.5 mm. With the closed helix just described placed tangential to the wave front the deflection was only 5 mm., and when the helix was turned through 90 degrees so as to have its plane normal to the wave-front the deflection was so small as to be scarcely readable. *The energy received by a closed helix apparently does not follow the same law as that of the open coil.* These results are somewhat surprising for if we assume the presence of a free Hertzian wave we would expect the open circuited helix to show a difference on orientation as well as the closed circuited coil.

SCREENED VERTICAL ANTENNA.

The foregoing experiments gave rise in the mind of the writer to the thought that possibly the greater portion, if not all, of the energy transmitted in wireless telegraph operations is propagated by means of electric oscillations *through the surface of the earth and not by means of the free ether surrounding the same.*

With this thought in mind the following experiment was devised. Two vertical antennæ each consisting of a single wire 6.2 m. in length were erected 17 cm. apart, and were so arranged at the base that either could be quickly coupled to the lead, *H*. One of these vertical antennæ was then shielded from the free Hertzian waves by placing about it a metal cylinder made of tin, the cylinder being 10 cm. in diameter and 6.2 m. long. The enclosed wire hung freely within the metal tube which was insulated from the earth. Resonance was separately effected for both open and enclosed antennæ. On two different occasions several series of readings were taken in alternate groups, the results for the two tests agreeing within less than .1 of 1 per cent. The shielded antenna produced a mean galvanometer deflection 9.3 per cent. less than the unscreened system. This is equivalent to a difference of 4.9 per cent. in the value of the

current in the two cases. The results of this experiment will be discussed elsewhere.

RELATIVE EFFICIENCY OF VARIOUS ENERGY-COLLECTING SYSTEMS.

We were now in a position to carry out a series of experiments designed to determine the relative efficiency as energy-gathering devices of the several types of antennæ now in commercial use. One or two original systems were also devised and tested. The two vertical single-wire antennæ described in the last experiment were so arranged as to permit of being connected at both the top and bottom, thus four different modifications of one system were possible. The metal cylinder used to enclose the vertical antenna of the last experiment was also suitably arranged to be used as a receiving system. The several cases are illustrated diagrammatically in Fig. 7 and will be referred to hereafter as systems *A*, *B*, *C*, *D* and *E*, as indicated in the drawing.

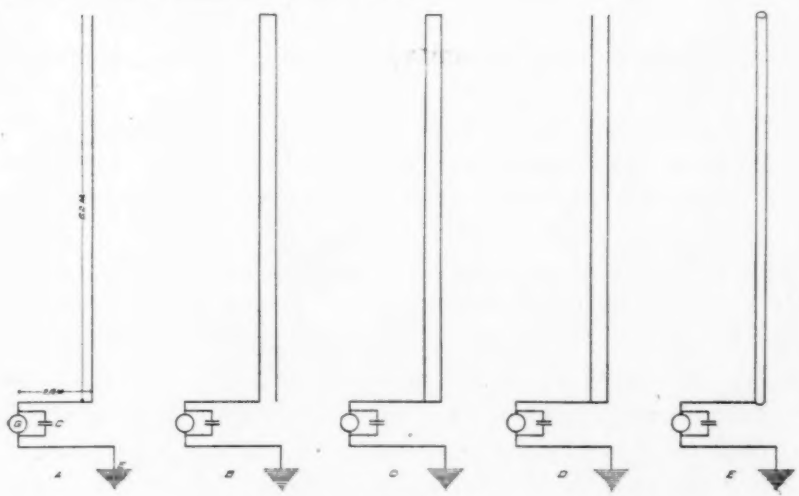

Fig. 7.

After the capacity[1] of the vertical portions of these systems was measured the systems were each in turn compared as energy-gathering devices with the simple vertical wire, *A*, taken as a standard. In these tests the readings for any given comparison were made by

[1] These and all other antenna capacity values were measured by the resonance-substitution method suggested in the paragraph on wave-length measurements.

taking several alternate groups of observations consisting of three or five readings each, thus eliminating the effects from any possible change at the sending station. (This method of taking readings was followed in all experiments constituting this investigation unless otherwise specified.) The mean ratios of the galvanometer deflection due to the energy received are to be seen in Table III., as are also the ratios of the current values in each case.

TABLE III.

System.	Capacity in Microfarads.	Ratio of Gal. Deflections Produced by Various Systems to that Caused by Standard System A.	Ratio of Current Values. (Sq. Root of Gal. Deflection.)
A	0.000032	1.000	1.000
B	0.000056	2.220	1.488
C	0.000068	2.045	1.450
D	0.000080	2.225	1.490
E	0.000060	3.520	1.880

It is apparent from an examination of Table III., that systems, *B* and *D*, are of practically equal efficiency. *C*, though very close to both *B* and *D* in value, showed a persistently lower efficiency. The difference is, however, only slight. It would appear that in erecting multiple-wired antennæ it is practically immaterial whether the component parts are connected at the lower, upper or both ends. Though the metal cylinder shows an apparently greater efficiency than the double-wire type, yet when other obvious factors are considered it is doubtful if such a system possesses any material advantage over the multiple-wire arrangement. The results would also seem to indicate that capacity is not as important a factor in determining the amount of energy received by a given system as is commonly supposed.

In the concluding series of experiments the relative efficiency of several systems already described, together with one or two original arrangements, were tested. The standard of comparison being a single vertical antenna 9.8 m. in length. The systems investigated in these tests were: a large capacity-area 4.2 × .7 m.; a small capacity-area; a large helix, 1.5 m. square, having 12 m. length of wire; a small helix, 15 cm. square, having 12 m. length of wire; a horizontal antenna, 9.8 m. in length; a wire screen, 1

m. square; a special system. The capacity-area, large helix and horizontal antenna were those used in previous experiments. The small helix was made by winding 11.1 m. of wire in 11 turns on a wooden frame 15 × 15 × 11 cm., .9 m. serving as a vertical lead to connect with the horizontal lead at the top of the supporting tripod. The small helix was so arranged that its center was at the same height above the ground as that of the larger helix. The inductance of the smaller coil was .056 milli-henrys. The wire screen consisted of a piece of wire netting, 1 m. square and having a mesh .5 cm. square, fastened to a wooden frame. As in the case of the small capacity-area a vertical wire .75 cm. long connected the middle of the screen to the horizontal lead. The special system consisted of 9.8 m. of wire fastened to a light wooden rectangular frame, in the manner indicated in Fig. 8.

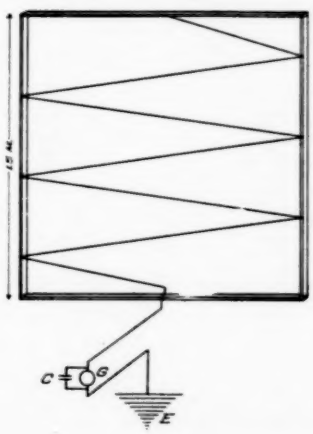

Fig. 8.

In all cases the systems were carefully insulated from the supporting structures by means of porcelain insulators. As in the tests on orientation, the large capacity and large helix were supported 70 cm. above the surface of the earth at the point of erection, and 16 cm. above the mean level between the two stations. The special system was supported in the same manner. The small capacity-area and the screen were 80 cm. and 25 cm. respectively above the surface and mean level. In the two last mentioned systems 1.5 m. of wire, nearly horizontal, led from the top of the supporting tripod to the lead, *H*. In the case of the large and small helixes and large capacity this connecting wire was 1.7 m. long. Readings were taken in the manner described in the previous experiment. The results are recorded in the following table.

An examination of the above table shows that *all of the receiving systems tested fell far short in efficiency in the matter of intercepting energy when compared with a single vertical wire.* The nearest approach to the vertical wire in the matter of efficiency is made by

TABLE IV.

System.	Inductance in Milli-henrys.	Capacity in Micro-farads.	Ratio of Gal. Def. Produced by Various Systems to that Caused by Standard.	Ratio of Current Values.
Standard vertical antenna.		0.000049	1.0000	1.0000
Large capacity-area.		0.000106	0.4060	0.6360
Small capacity-area.		0.000008 (?)	0.1750	0.4190
Large helix.	0.041	0.000044	0.2520	0.5020
Small helix.	0.056	0.000034	0.1060	0.3260
Horizontal antenna.		0.000340	0.1730	0.4160
Wire screen.		0.000044	0.1970	0.4440
Special system.		0.000030	0.2290	0.4790

the large capacity-area which has an efficiency in current value of a trifle over 63 per cent. It is to be noted, however, that this system has a capacity of more than twice the value of that of the standard antenna. This fact bears out the statement previously made in reference to the effect of capacity in a receiving system. It may also be observed that the horizontal antenna, at least when placed comparatively near the earth, has a relatively small efficiency. The small capacity-area, though only 1 m. square, shows a slightly greater efficiency than the horizontal system, and the wire screen is even still more efficient relatively. The case of the large and small helixes is also of interest. The smaller coil, though having the same length of wire as the larger shows a material falling off in energy received when compared with the larger. This difference may possibly be due to the greater impedance of the smaller helix. When comparing both large and small helixes with the standard type, it is to be remembered that these coils were each made up of 12 m. of wire, while the single wire standard consisted of only 9.8 m. From a practical point of view when length of wire, facility of erection and ease of maintenance are considered the special type shows the greatest efficiency as a receiving system. Its efficiency would probably have been materially increased had its lengths of wire been made to run vertical instead of nearly horizontal.

The resonance curve shown in Fig. 9 above is inserted to show the general properties of the two waves sent out by the sending station. The data for this curve was taken when using the capacity-

area 1 m. square as a receiving system. Capacity shunted about the galvanometer is plotted against galvanometer deflections. The lesser maximum probably corresponds to the shorter of the two wave-lengths radiated by the transmitting station and the greater

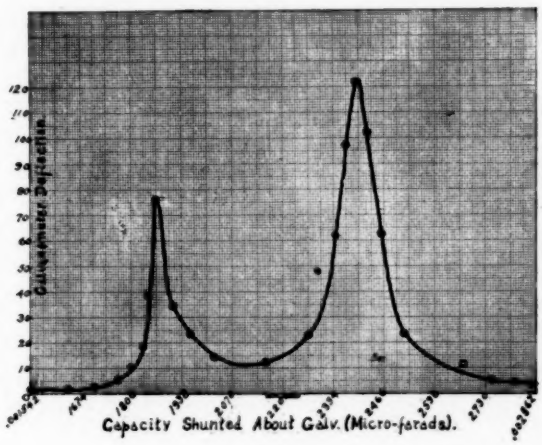

Fig. 9.

maximum to the longer wave-length. However, this relation is somewhat difficult to determine, because of the fact that these maxima would frequently interchange places. The reason for this rather unusual phenomenon we have as yet been unable to determine.

Results.

To recapitulate, the foregoing data would indicate, under the conditions which obtained in these experiments: (1) that the resistance of the earth between the two stations is an important factor in the propagation of energy; (2) that the square of the energy received by a horizontal antenna is approximately proportional to its length; (3) that relatively small capacity-areas show equal efficiency in all planes of orientation about a vertical axis; (4) that even in the case of capacity-areas whose length is great compared with the width only a slight decrease in efficiency is noticeable when the area is normal to the wave front: (5) that open-circuited helixes are equally efficient in all planes of orientation; (6) that the energy received by such a helix having a given length of wire is a function

of its dimensions; (7) that an aërial almost completely screened is but slightly less efficient than a similar unscreened system; (8) that in dealing with multiple-wired antennæ it is practically immaterial as to whether the component parts are connected at the lower, upper or both ends; (9) that in considering different types of receiving systems the actual capacity is not so important as is the manner in which this capacity is distributed; (10) that of the types tested the system consisting of one or more wires normal to the earth's surface is by far the most efficient; (11) that the energy is propagated through the surface of the earth and not by means of a free ether wave.

DISCUSSION.

The comparative closeness of our stations afforded us a special opportunity to study the effects due to the free Hertzian waves as differentiated from the effects due to the propagation of energy in other possible ways. Stone[1] maintains that the greater portion of the energy transmitted resides in that part of the wave which is immediately adjacent to the surface of the earth. It will be remembered in this connection that our capacity-areas were placed very close to the surface of the earth. If the free wave ever enters as an important factor it should, according to the commonly accepted view, have produced marked effects in our experiments, but, on the contrary, we find in the cases where the capacity-areas, helixes, etc., were rotated about a horizontal axis variation in the plane of orientation produced but slight changes in the amount of energy received, and in some instances resulted in no change whatever. The complete screening of the vertical antenna reduced the received energy but a small per cent. and even this change may have been due to causes other than the shielding effects due to the metal enclosure.

In the experiments having to do with earth resistances the increase in the quantity of energy received was probably not wholly due to the decrease in the resistance between the two stations, but it is reasonable to suppose that the greater part of the increase may be ascribed to this cause.

In view of the facts just enumerated, and since a fixed wave-length and constant supply of energy was radiated from the trans-

[1] Stone, Transactions International Electrical Congress, St. Louis, 1904, p. 558.

mitting station, it would seem that the conclusion is not wholly unwarranted that practically all the energy taking part in wireless telegraph operations is propagated through the surface of the earth by means of electrical oscillations and not by means of free Hertzian waves.

The experience of those engaged in practical wireless work bears out the conclusion of the author in this respect. It is well known that when working over land a slight fall of rain on dry soil materially increases the signalling distance. The fact that it is possible to signal through greater distances over sea than over land we believe to be due to the greater conductivity of the water. The originator of one of the wireless systems now in commercial use informs the writer that in some recent tests made it was found that when operating overland the presence of iron bearing ore in the earth materially increased the signalling distance. Neither the author's experience nor the instance just cited bear out the contention of Sacks[1] who maintains that any good conductor in, or in connection with, the earth is detrimental in its effects. Kimura[2] also holds that the earth resistance does not figure as a factor but that the capacity of the earth plate is the important consideration. The above commercial and theoretical experiments do not tend to confirm this view.

Again, if the theory of a free ether wave is correct, the law of inverse squares should hold, approximately at least. That this law *does not hold* has been shown by several investigators among whom may be mentioned Duddell and Taylor,[3] who found that the product of the distance by the intensity was a constant, and Chant,[4] who also found that the energy fell off inversely as the simple distance.

In considering the possible processes by which energy is propagated in wireless telegraphy it is to be remembered that the beginning of wireless telegraphy as a practical commercial project dates from the time when Marconi first *connected his transmitting apparatus to earth.* The distances over which communication was effected prior to this were insignificant. In fact, until that time the transmission

[1] J. S. Sacks, Ann. der Physik, Vol. 18, p. 348.
[2] Kimura, Phys. Zeitcher, June 29, 1901.
[3] Institute Electrical Engineers Journal, Vol. 35, p. 321.
[4] American Journal of Science, Vol. 18, p. 403.

of intelligence by means of electric waves was little more than an interesting laboratory experiment. In a paper read before the International Electrical Congress, which convened at St. Louis, 1904, Count Solari made the following significant statement: "In other words he (Marconi) made the great discovery that two rods of metal placed *upright in the ground* at some distance apart *form a gigantic and novel oscillator in which electric oscillations set up in the one part are propagated through the earth to the other part*,[1] and at the same time electrical waves formed by the alternations of electric strain directed perpendicularly to the earth and the associated magnetic forces parallel to the earth, are propagated through the ether between the two vertical wires." We hold that the first factor mentioned by the authority just quoted constitutes the vital consideration, and that it was due to this principle that Marconi succeeded in making long distance transmission possible. We do not dispute the existence of the free ether wave but maintain that its effect is nill beyond comparatively short distances from the radiating system.

May we not think of a wireless transmitting apparatus when in operation as impressing upon the earth, at the point where the system is grounded, a high frequency alternating potential, these waves of potential spreading out through the surface of the earth in all directions? If we suppose such a process as taking place, another conductor placed in a vertical position at a distant point and connected either directly or inductively with the surface of the earth would be charged to the oscillating potential represented by the electric earth-wave at that point. Thus we would have the energy transmitted to the receiving system. The resistance of the earth would doubtless diminish the amplitude of the potential wave, hence the importance of high earth conductivity between two stations. It is more than probable that such a potential wave is confined to a very thin stratum of the earth's surface as is maintained by Poincare.[2]

We are aware that at least one empirical fact is difficult of explanation on the basis of the theory which we advance. Professor Fleming quotes the observations of Captain Jackson, of the

[1] The italics are ours.

[2] H. Poincare, Bureau des Long. Ann., 1902.

British Navy, relative to the effect of intervening land upon signalling distance, the observation being that hilly or stony ground intervening between two stations greatly decreases the amount of energy transmitted, thus reducing the signalling distance. Lieutenant-commander Robinson of the United States Navy informs the writer that it is a common experience in the Navy to have signals cease almost instantly when an elevated body of land comes to intervene between two vessels or a vessel and a land station. Lieutenant Robinson made the significant remark that "Every hill casts its shadow." We grant this, but we believe that the sudden decrease in energy when passing behind elevated bodies of land can be accounted for on other grounds than that of the electrical shadow in the commonly accepted sense of the term. The author is at present designing a series of experiments by which he hopes to prove this contention.

The phenomenon of the effects of light and darkness upon the propagation of energy we admit is not readily explicable on the assumptions which we advance. However a number of important facts which are difficult to account for upon the idea of a free Hertzian wave become easy of explanation on the basis of the oscillation theory, one notable example being the case of Marconi's Trans-Atlantic communication. This is difficult to explain on the basis of a free ether wave, as is cited by Poincare,[1] but presents no particular difficulty when viewed in the light of the theory outlined above. Notwithstanding the failure of the proposed theory to account for several phenomena encountered in practical wireless work, we believe such a theory accounts for more of the facts now known than the conventional idea of a free energy wave.

RANDAL MORGAN LABORATORY,
UNIVERSITY OF PENNSYLVANIA,
May 1, 1907.

[1] H. Poincare, Proc. Roy. Soc., Vol. 72, July 8, 1903.

NEW BOOKS.

Electricity Meters: Their Construction and Management. By C. H. W. Gerhardi. Pp. i + 326. London, The Electrician Printing and Publishing Company, 1906.

This book purports to be a practical manual and as such is eminently successful. Physical theory and mathematical discussion are passed by and the whole aim of the book has been to meet the needs of central station managers, engineers and students who wish full information in regard to the installation, use and maintenance of meters. Meter testing is discussed in great detail, and included with it is a complete description of meter testing equipment. The book is written primarily from a British standpoint, the author being chief of the Testing Department of the Metropolitan Electric Supply Company, London. It is the counterpart of American Meter Practice by L. C. Reed. The treatment is much more complete.

F. BEDELL.

Graphical Treatment of the Induction Motor. By ALEXANDER HEYLAND. Translated by G. H. ROWE and R. E. HELLMUND. Pp. i + 48. New York, The McGraw Publishing Company, 1906.

On account of the important place now taken by the Heyland Diagram in the study and tests of induction motors, this translation of Heyland's original work will be much appreciated. One is impressed with the simplicity and clearness of exposition with which the subject is developed. Although the subject has been amplified and more elaborately treated by recent writers, the student might very well peruse this little book before taking up more extensive treatises.

F. BEDELL.